MAY 3 1929

VOLUME LXIX NUMBER 2

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

MARCH 1929

THE BAND SPECTRUM OF FLUORINE - - - *H. G. Gale and G. S. Monk* 77

A SPIRAL NEBULA AS A STELLAR SYSTEM, MESSIER 31 - - - *Edwin Hubble* 103

ON THE HELIOCENTRIC VELOCITY OF METEORS - - - *C. Hoffmeister* 159

REVIEWS

Analyse periodischer Vorgaenge, KARL STUMPFF (B. P. Gerasimović), 168.—*Ergebnisse der exakten Naturwissenschaften*, JULIUS SPRINGER (ed.), 170.—*Die Seltenen Erden vom standpunkte des Atombaues*, G. Hevesy, (L. F. Yntema), 171.—*Amateur Telescope Making*, ALBERT G. INGALLS (S. B. B.), 172.

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

March 1929 Vol. LXIX, No. 2

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

WITH THE COLLABORATION OF

WALTER S. ADAMS, Mount Wilson Observatory
JOSEPH S. AMES, Johns Hopkins University
ARISTARCH BELOPOLSKY, Observatoire de Pulkovo
WILLIAM W. CAMPBELL, Lick Observatory
HENRY CREW, Northwestern University
CHARLES FABRY, Université de Paris
ALFRED FOWLER, Imperial College, London
CHARLES S. HASTINGS, Yale University
HEINRICH KAYSER, Universität Bonn
ALBERT A. MICHELSON, University of Chicago
ROBERT A. MILLIKAN, Institute of Technology, Pasadena
HUGH F. NEWALL, Cambridge University
HENRY N. RUSSELL, Princeton University
FRANK SCHLESINGER, Yale Observatory
SIR ARTHUR SCHUSTER, Twyford
FREDERICK H. SEARES, Mount Wilson Observatory

The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶ The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, Canary Islands, El Salvador, Argentina, Bolivia, Brazil, Columbia, Chile, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Hayti, Uruguay, Paraguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Balearic Islands, Spain, and Venezuela. ¶ Postage is charged extra as follows: for Canada and Newfoundland, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies 5 cents (total 80 cents). ¶ Patrons are requested to make all remittances payable to The University of Chicago Press, in postal or express money orders or bank drafts.

The following are authorized agents:

For the British Empire, except North America, India, and Australasia: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 12s. 6d. each; single copies, including postage, 4s. each.

For Japan: The Maruzen Company, Ltd., Tokyo.

For China: The Commercial Press, Ltd., Paoshan Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business Correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "Observatory, Williamsbay, Wisconsin."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895 at the Post-office at Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXIX MARCH 1929 NUMBER 2

THE BAND SPECTRUM OF FLUORINE

BY H. G. GALE AND G. S. MONK

ABSTRACT

New measurements show that the bands of this spectrum consist of P, Q, and R branches with *alternate weak and strong lines* in each branch. The Q branch is about twice as strong as the P and R branches. Two of the strongest bands, whose null points lie at $\nu_0 = 16378.8$ and $\nu_0 = 17439.5$, have been completely analyzed, and three fainter bands at $\nu_0 = 15337.6$, $\nu_0 = 17075.9$, and $\nu_0 = 18548.4$ partly analyzed. There are *strong perturbations in all the bands* in both frequency and intensity, which have hitherto masked their true character. These perturbations are in the initial state. The analysis of the bands yields the following values:

$$B_0' = 0.815, \quad B_0'' = 1.073, \quad I_0' = 34.2 \times 10^{-40},$$
$$I_0'' = 26.0 \times 10^{-40}, \quad \omega_0' = 836.83, \quad \omega_0'' = 1130.1,$$
$$\omega_0' x' = 140.6, \quad \omega_0'' x'' = 9.7, \quad a'' = 0.014.$$

The structure and intensity relations indicate that the bands are of the $^1S \rightarrow {}^1P$ type. Hund has shown that a ratio of intensities of strong to weak lines within a branch of 3:1 indicates a *nuclear spin of* $\frac{1}{2}(h/2\pi)$. The best plates show this intensity ratio to be at least 3:1 and probably greater.

The band spectrum obtained in a discharge tube through which fluorine gas is flowed was first photographed by W. R. Smythe[1] and for experimental reasons attributed by him to CF_4. Later, high-dispersion photographs were obtained by the authors, and their appearance reported.[2] Owing to the extreme difficulty of securing plates of sufficient intensity with high dispersion, only a partial analysis was then made, the substance of which was chiefly the arrangement of the heads of the bands into a scheme offering the most satisfactory assignment of vibrational quantum numbers for

[1] *Astrophysical Journal*, **54**, 133, 1921.

[2] *Ibid.*, **59**, 125, 1924.

the different bands.[1] At the time of this last report, no satisfactory analysis of the fine structure of the bands had been made. It is evident, therefore, that the assignment of the vibrational quantum numbers lacked the support of the evidence obtainable from the application of the combination principle to the fine structure.

Since then, through repeated trials, we have been successful in obtaining comparatively good photographs with large dispersion (2.63 A per millimeter) of the three strongest bands in the system. Together with the plates previously obtained, these have enabled us to make the analysis presented in the following pages.

APPARATUS

Since the experimental conditions are the basis of assumptions concerning the origin of the spectrum, it seems best to describe the apparatus again. The fluorine was prepared by passing an electrolyzing current of 2 or 3 amp. through KHF_2 contained in a copper vessel at a temperature of about 245° C. The vessel served as the cathode and a rod of hard carbon or graphite as the anode. The fluorine rising from the anode was collected in a copper cylinder surrounding the carbon and insulated from it, and was passed first through a tube of fused NaF, then through a trap immersed in frozen ethyl alcohol (melting point, −114° C.), in order to remove the HF which might have been present. The control of the pressure of the fluorine gas passing through the discharge tube was accomplished by means of a needle valve made of copper, and a finely drawn capillary tube in series, placed between the frozen alcohol trap and the discharge tube. Beyond the discharge tube, and some feet away, was a tube containing charcoal through which the gas was passed before it entered the pump, by means of which the discharge tube was kept at a pressure of about 5 mm.

During the earlier part of the investigation tests were made to determine experimentally the probable source of these bands. It was thought at first that they might be due to CF_4 because of the nearness of the charcoal tube to the discharge tube. But the removal of this tube made no difference in the presence of the bands. In any case it is doubtful whether a complicated molecule such as CF_4 could

[1] *Physical Review*, **29**, 211, 1927.

give rise to a system of bands having the structure of those obtained. Of course there remained the possibility that the bands are due to CF_4, originating from combination of the fluorine with the carbon anode. It will be seen from the discussion of the fine structure that this could hardly be the case. The bands were not found in the spectrum obtained with a discharge tube through which *HF* was flowed continuously.

Furthermore, liquid air was substituted for frozen alcohol in the trap between the generator and the discharge tube, with no effect on the presence of the bands.[1] It is doubtful whether *C-F* compounds could have passed the trap, were they present. In a more recent experiment the discharge tube and a foot or two of tubing leading away from it on both sides were inclosed in a liquid-air trap. The purpose of this particular arrangement was to see if certain of the band lines could be sharpened and their resolution thereby facilitated, but it is mentioned here as additional evidence that the bands are not due to CF_4.[2] The bands also appear unchanged despite the use of different substances for the electrodes or the use of external electrodes.

The two necessary conditions for the appearance of the bands are (1) a continuous supply of fresh fluorine from the generator and (2) a discharge of low current density. If the fluorine supply is for any reason cut off, the bands disappear and give way to the spectrum of nitrogen. The presence of nitrogen has never been completely eliminated, and unless it diffuses back from the exhaust end of the system it is difficult to understand its presence. It is believed not to originate from slight leaks in the generator or needle valve since the pressure in the generator is maintained a centimeter or two higher than atmospheric pressure; likewise that part of the needle valve in which the valve is located is on the side of high pressure.

If the discharge is too strong, the fluorine lines remain but the bands disappear from the capillary, although they remain in the

[1] The boiling-point of nitrogen is $-195°$ C.; that of oxygen, $-182°.7$ C.; that of fluorine, $-187°$ C. It was necessary to keep fresh liquid air about the traps in order to collect any appreciable amount of liquid fluorine (see *Astrophysical Journal*, **59**, 125, 1924).

[2] The bands of SiF_4 are not present.

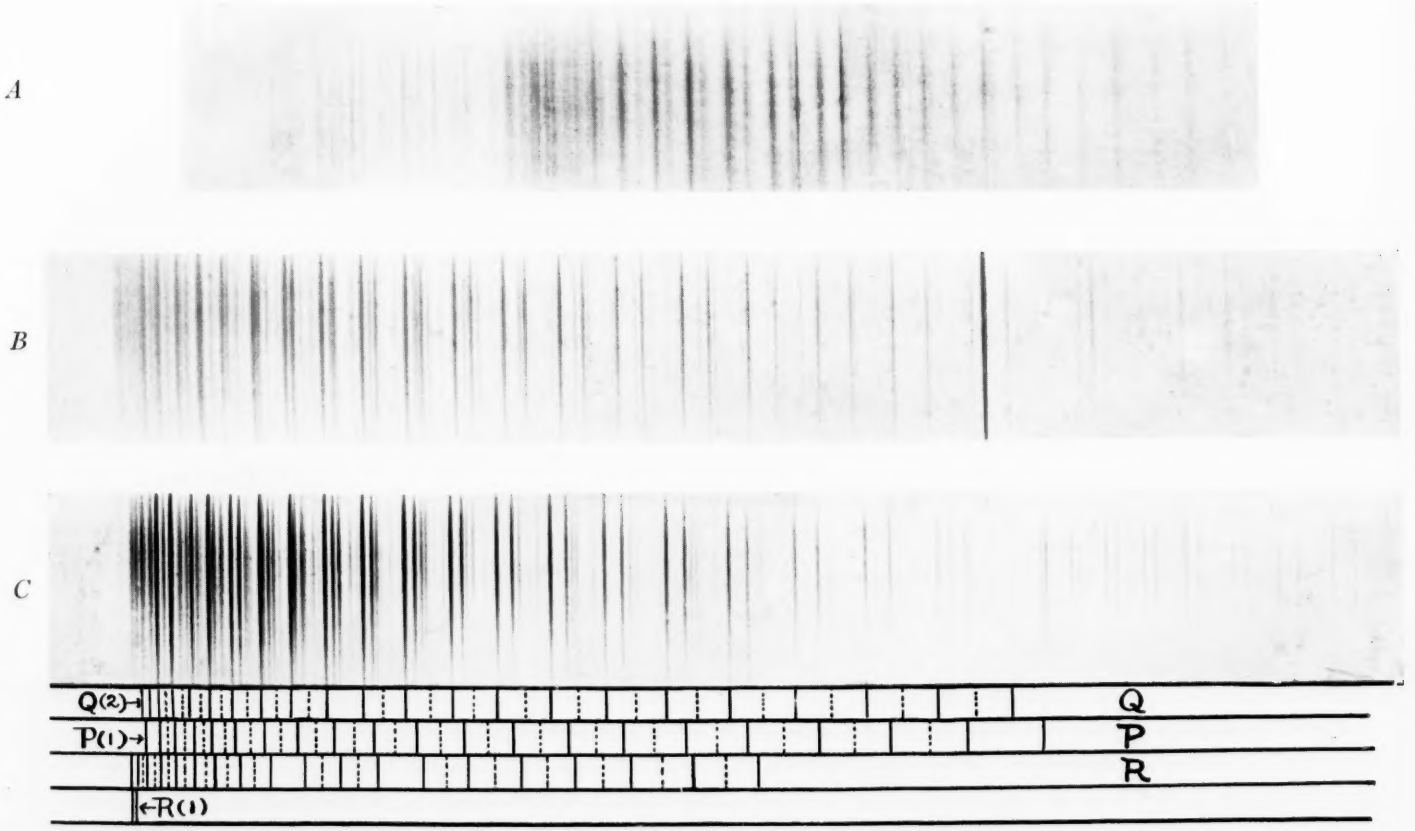
larger portions of the discharge tube. In the capillary they are replaced by the *CO* bands, silicon lines, and many other impurities evidently due to decomposition of the glass by the fluorine. For this reason, in later experiments the capillary was replaced by a tube of 5- or 6-mm inside diameter, but even then the current has to be restricted to 10 or 15 milliamperes.

The light was taken from the end of the discharge tube which was covered by a fluorite window. Although this window was not attacked by the fluorine, it invariably became coated after four or five hours with flaky deposit due to the decomposition of the glass. Attempts to remove the window and clean it and continue the exposure nearly always met with disaster because of the condition of all the glass in the system. An exposure of more than eight hours, during which the fluorine spectrum was absent part of the time while the generator was being nursed back to maximum productivity, was rare.

MEASUREMENT OF THE SPECTRA

The spectrum of iron was used as a standard of comparison, and the plates were measured to thousandths of a millimeter on a Gaertner comparator. For the stronger lines, good agreement was obtained between the different plates. The wave-lengths of the fainter lines were obtained from two plates only, and a few from only one. It is estimated that the measurements of wave-lengths of all except the faintest lines are accurate to 0.05 A. This does not mean necessarily that all the wave-numbers shown in the following tables are accurate to that degree, since a certain element of arbitrariness enters into the assignment of overlapping lines to various branches, particularly near the head of each band. For the band at $\nu_0 = 15337.6$ the foregoing degree of accuracy is by no means obtained since practically every line in the band is over 0.5 A wide and very fuzzy, with much overlapping of lines.

The two bands yielding most easily to analysis are $\nu_0 = 17439.5$ and $\nu_0 = 16378.8$. Photographs of these bands are shown in Plate II. It will be seen that each band seems to consist of a series of doublets, one component of each somewhat stronger than the other, with many fainter lines not so obviously connected in typical



A, Band with null point at 15337.6
B, Band with null point at 16378.8
C, Band with null point at 17439.5, with analysis of fine structure below. To the right is the band at 17075.9

branches. In fact, all the bands of the system have this characteristic appearance. It is easy to arrange the stronger lines into two branches, and fairly easy to identify a third branch, but in addition there are many fainter lines having no easily recognizable sequence as members of additional branches. Since, however, all the observational evidence tends to indicate that the emitter of the bands is F_2, it was realized that, in common with the Runge bands of oxygen, and bands of other symmetrical di-pole molecules, there should either be a set of branches with alternate lines completely missing or else with alternate lines much weakened. An attempt to utilize the fainter lines, by assigning them to positions on the three identified branches, at first met with no success. The principal reason for this is that not only do the second differences between the lines, alternately weak and strong, depart entirely from constancy, but there are in some cases complete gaps in the branches. But because it was evident that we were here dealing with bands having strong perturbations in both intensity and wave-number, the frequencies of bands $\nu_0 = 17439.5$ and $\nu_0 = 16378.8$ were put into branches containing alternate weak and strong lines, with the most likely assignment of weak lines where overlapping, proximity to other lines, or perturbations made correct assignment difficult. It at once became apparent that each band contained three branches, a strong Q, a weak P, and a still weaker R branch. Certain characteristic perturbations in these branches made it possible to assign proper j-values to the branches (rather, correct values differing from j by a constant) on the hypothesis that the perturbations are connected with the initial state. It was found that with these assignments the combination relation $R_j - Q_{j+1} = Q_j - P_{j+1}$ held approximately. Since the characteristic perturbations occur in several bands, including the two under discussion, it was assured that they belong to the same n''-progression. In accordance with the requirements of the combination principle, $R_j - P_j = \Delta_2 F'(j)$ was found to be the same for the two bands. Tables I and II give the wave-numbers, intensities, and first and second differences for the two bands. Reference to the illustrations in Plate II will show that near the head of the band it is extremely difficult to identify the lines of P, Q, and R branches. This difficulty is increased by the perturbations. There are, however, two

TABLE I

Wave-Numbers and Intensities

Band at $\nu_0 = 17439.5$

j*	R Branch				Q Branch				P Branch			
	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$
41....	2	17150.41			2	17084.00			1	17018.54		
			16.17				16.57					
40....	1	17166.58		2.24	5	17100.57		0.41				
			13.93				16.16					
39....	3	17180.51		−0.34	3	17116.73		0.12	1	17055.10		
			14.27				16.04				17.08	
38....	2	17194.78		0.27	5	17132.77		0.75	1	17072.18		−0.12
			14.00				15.29				17.20	
37....	4	17208.78		0.22	4	17148.06		0.16	2	17089.38		0.62
			13.78				15.13				16.58	
36....	2	17222.56		0.98	10	17163.19		0.53	1	17105.96		0.36
			12.80				14.60				16.22	
35....	5	17235.36		−0.05	5	17177.79		0.27	2	17122.18		0.42
			12.85				14.33				15.80	
34....	2	17248.21		0.70	13	17192.12		0.57	2	17137.98		−1.01
			12.15				13.76				16.81	
33....	7	17260.36		0.56	5	17205.88		0.91	3	17154.79		2.54
			11.59				14.67				14.27	
32....	1	17271.95		−1.35	15	17220.55		2.18	1	17169.06		−0.28
			12.94				12.49				14.55	
31....	8	17284.89		2.21	6	17233.04		−0.30	5	17183.61		0.76
			10.73				12.79				13.79	
30....	2	17295.62		0.28	20	17245.83		1.23	1	17197.40		−1.37
			10.45				11.56				15.16	
29....	7	17306.07			3	17257.39		−1.78	5	17212.56		2.25
							13.34				12.91	
28....	....	.....			25	17270.73		2.44	2	17225.47		0.03
							10.90				12.88	
27....	9	17326.98			9	17281.63		−0.31	6	17238.35		−0.49
			9.01				11.21				13.37	
26....	2	17335.99		0.19	30	17292.84		0.16	3	17251.72		1.42
			8.82				11.05				11.95	
25....	12	17344.81		−0.63	7	17303.89		0.78	11	17263.67		−1.08
			9.45				10.27				13.03	
24....	1	17354.26		1.39	35	17314.16		−0.21	1	17276.70		2.45
			8.06				10.48				10.58	
23....	12	17362.32			2	17324.64		1.56	12	17287.28		−2.61
							8.92				13.19	
22....	....	.....			38	17333.56			<1	17300.47?		4.50
											8.69	
21....	16	17378.18			....	.....			12	17309.16		−2.67
			6.51								11.36	
20....	1	17384.69		−0.41	40	17351.40			4	17320.52		2.53
			6.92				9.40				8.83	
19....	21	17391.61		−0.77	4	17360.80		2.52	13	17329.35		−1.00
			7.69				6.88				9.83	
18....	1	17399.30		2.84	45	17367.68		−0.87	2	17339.18		1.41
			4.85				7.75				8.42	
17....	20	17404.15		−0.68	12	17375.43		1.19	13	17347.60		−0.68
			5.53				6.56				9.10	
16....	2	17409.68		1.19	45	17381.99		−0.12	2	17356.70		0.25
			4.34				6.68				8.85	
15....	14	17414.02		−0.44	9	17388.67		−0.46	12	17365.55		1.90
			4.78				7.14				6.95	
14....	2	17418.80		−0.46	40	17395.81		2.21	2	17372.50		−1.26
			5.24				4.93				8.21	
13....	10	17424.04		2.42	7	17400.74		−1.01	12	17380.71		1.49
			2.82				5.94				6.72	
12....	2	17426.86		−1.64	35	17406.68		0.73	2	17387.43		−1.35
			4.46				5.21				8.07	
11....	7	17431.32		2.48	6	17411.89		0.62	15	17395.50		2.83
			1.98				4.59				5.24	
10....	2	17433.30		−0.85	28	17416.48		1.07	3	17400.74		0.00
			2.73				3.52				5.24	
9....	6	17436.03		0.88	5	17420.00		−0.52	13	17405.98		−0.28
			1.85				4.04				5.52	

* Here, as in R_j, P_j, etc., j means j''.

TABLE I—*Continued*

j	R Branch				Q Branch				P Branch			
	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$
8....	2	17437.88		−0.50	25	17424.04		0.50	2	17411.50		2.84
			2.35				3.54				2.68	
7....	4	17440.23			4	17427.58		0.60	10	17414.18		−2.91
							2.94				5.59	
6....	....	.....			17	17430.52			2	17419.77		1.32
											4.27	
5....	3	17441.60			....	.....			8	17424.04		0.93
											3.34	
4....	....	.....			8	17434.65			2	17427.38		−0.73
							3.38?				4.07	
3....	3	17442.79			1?	17436.03			5	17431.45		
							1.97?					
2....	....	.....			2	17438.00			....	.....		
1....	1	17442.01			....	.....			1	17437.20		

rules which make this identification possible: (1) The quantities $R_j - P_j = \Delta_2 F'(j)$ are the same for different bands with the same initial vibrational state; (2) for the strong lines of the branches, at least, the total intensity of a composite line should be properly accounted for. It was found that not only were the intensities properly accounted for, but that practically every line on the plate in each of these two bands is included in the analysis. Table III shows for these two bands the values of $R_j - Q_j$ and $Q_j - P_j$. Columns 4 and 7 show good agreement. The values in column 2 must be compared with those one line higher in column 3, in accordance with the rule $R_j - Q_j = Q_{j+1} - P_{j+1}$; likewise in columns 5 and 6 for the second band.

THE QUANTUM DEFECT

On account of the scarcity of measurements of the faint alternate lines for all but two of the strong bands, it is not possible to give very extensive data for the quantum defect. For the two strong bands the differences $(R_j - Q_j) - (Q_{j+1} - P_{j+1})$ have been averaged for each j-value and the points plotted in Figure 1. Van Vleck states that the quantum defect should be proportional to $j(j+1)$.[1] Fitting the points observed to this curve, the dotted line in the figure is obtained. Although there are not enough data to differentiate between possible forms of this curve, the evidence is presented to indicate the extent to which the Van Vleck formula is fitted.

In general, it will be seen that the second differences for the two

[1] See also R. L. de Kronig, *Zeitschrift für Physik*, **50**, 356, 1928.

TABLE II

WAVE-NUMBERS AND INTENSITIES
Band at $\nu_0 = 16378.8$

j	R BRANCH				Q BRANCH				P BRANCH			
	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$
42....	1	16093.18			2	16027.20			1	15960.54		
			17.08				16.36				17.86	
41....	4	16110.26		2.34	1	16043.56		0.92	1	15978.40		0.55
			14.74				15.44				17.31	
40....	1	16125.00		1.60	3	16059.00		0.08	1	15995.71		0.42
			13.14				15.36				16.89	
39....	4	16138.14		−0.39	2	16074.36		0.23	2	16012.60		0.26
			13.53				15.13				16.63	
38....	3	16151.67		0.55	5	16089.49		0.15	2	16029.23		0.78
			12.98				14.98				15.85	
37....	5	16164.65		0.15	2	16104.47		1.42	3	16045.08		−0.03
			12.83				13.56				15.88	
36....	3	16177.48		0.94	7	16118.03		−0.17	2	16060.96		0.77
			11.89				13.73				15.11	
35....	7	16189.37		0.01	3	16131.76		0.23	4	16076.07		0.28
			11.88				13.50				14.83	
34....	3	16201.25		0.43	10	16145.26		0.54	1	16090.90		−1.11
			11.45				12.96				15.94	
33....	7	16212.70		0.65	3	16158.22		−0.92	5	16106.84		2.20
			10.80				13.88				13.74	
32....	3	16223.50		−1.33	15	16172.10		2.15	2	16120.58		0.06
			12.13				11.73				13.68	
31....	9	16235.63		2.60	4	16183.83		−0.20	6	16134.26		0.94
			9.53				11.93				12.74	
30....	3	16245.16		−0.63	18	16195.76		0.72	1	16147.00		−2.01
			10.16				11.21				14.75	
29....	8	16255.32		0.66	2	16206.97		−1.07	10	16161.75		2.33
			9.50				12.28				12.42	
28....	4	16264.82		−0.62	25	16219.25		1.90	4	16174.17		0.48
			10.12				10.38				11.94	
27....	8	16274.94		1.79	3	16229.63		0.37	8	16186.11		−0.91
			8.33				10.01				12.85	
26....	4	16283.27		−0.05	30	16239.64		−0.98	2	16198.96		0.42
			8.38				10.99				11.43	
25....	8	16291.65		−0.33	6	16250.63		* 1.64	11	16210.39		−0.80
			8.71				9.35				12.23	
24....	6	16300.36		0.34	35	16259.98		−0.73	5	16222.62		2.06
			7.37				10.08				10.17	
23....	10	16307.73			1	16270.06		1.73	12	16232.79		
							8.35					
22....	....	.....			42	16278.41			....	.....		
21....	12	16322.01			....	.....			15	16253.19		
			6.25								10.99	
20....	3	16328.26		−0.37	45	16295.23			5	16264.18		2.77
			6.62				8.67				8.22	
19....	8	16334.88		−0.34	3	16303.90		2.05	15	16272.40		−1.38
			6.96				6.62				9.60	
18....	5	16341.84		2.80	48	16310.52		−0.84	8	16282.00		1.54
			4.76				7.46				8.06	
17....	10	16346.60		−0.51	10	16317.98		1.47	18	16290.06		0.62
			5.27				5.99				8.68	
16....	6	16351.87		1.82	45	16323.97		−0.49	4	16298.74		−0.01
			4.05				6.48				8.69	
15....	6	16355.92		−0.25	11	16330.45		−0.13	15	16307.43		2.30
			4.30				6.61				6.39	
14....	3	16360.22		−0.52	38	16337.06		1.83	3	16313.82		−1.80
			4.82				4.78				8.19	
13....	5	16365.04		2.26	9	16341.84		−1.55	18*	16322.01		1.94
			2.56				6.33				6.25	
12....	3	16367.60		−0.64	32	16348.17		3.30	3	16328.26		−0.17
			3.20				3.03				6.42	
11....	3	16370.80		1.60	5	16351.20		−1.68	15	16334.68		1.10
			1.60				4.71				5.32	
10....	1	16372.40		−2.48	25	16355.91		0.00	2	16340.00		−0 78
			4.08				4.71				6.10	
9....	2	16376.48		2.36	5	16360.62		0.53	12	16346.10		0.38
			1.72				3.18				5.72	

* This line is not separate from R(21).

TABLE II—*Continued*

j	R Branch				Q Branch				P Branch			
	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$
8....	2	16378.20		0.12	20	16363.80		−0.65	3	16351.82		3.36
			1.60				3.83				2.36	
7....	4	16379.80			5	16367.63		1.28	13	16354.18		−3.06
							2.55				5.42	
6....	....	.....			13	16370.18		0.53	3	16359.60		0.42
							2.02				5.00	
5....	4	16381.60			1	16372.20		0.18	8	16364.60		
							1.84					
4....	....	.....			8	16374.04			....	.....		
3....	3	16381.90			....	.....			6	16370.80		
2....	....	.....			4	16373.22			....	.....		
1....	1	16381.20			....	.....			1	16376.42		

bands are widely divergent, indicating the character of the perturbations in frequency which caused so much difficulty in the analysis. But except for the region near the heads of the bands where there is much overlapping there is good agreement between the second

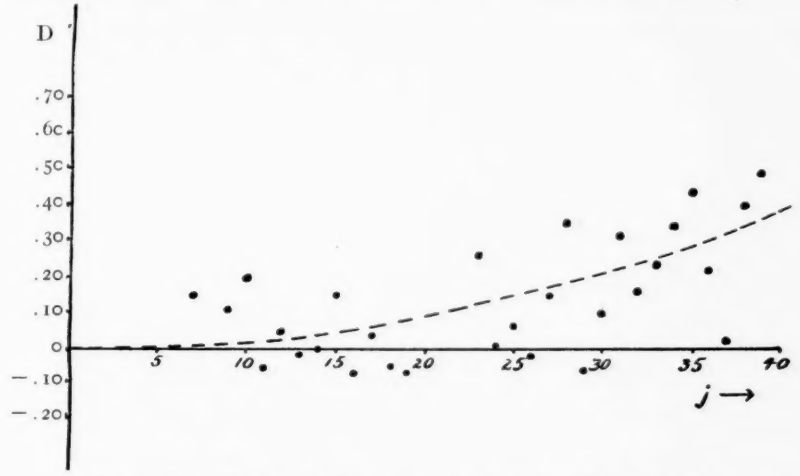


FIG. 1.—The quantum defect averaged for the 0,3 and 0,4 bands

differences in the R, Q, and P branches for each band. There is evidence, in the agreement between corresponding branches of the two bands, that these belong to the same initial state. The gaps in the tables for small j-values are due to the very small intensities of the weak alternate lines, whose strength reaches a maximum farther out in the branches. The weakness of these lines near the head is complicated by the overlapping of the branches. A curious condi-

tion in the alternate weak lines is found for j-values in the neighborhood of $j=22$, where there is a line missing, or at least not of suffi-

TABLE III

j	17439.5			16378.8		
	R_j-Q_j	Q_j-P_j	R_j-P_j	R_j-Q_j	Q_j-P_j	R_j-P_j
42	.....	.....	.....	65.98	66.66	132.64
41	66.41	65.46	131.87	66.70	65.16	131.86
40	66.01	.....	.....	66.00	63.29	129.29
39	63.78	61.63	125.41	63.78	61.76	125.54
38	62.01	60.59	122.60	62.18	60.26	122.44
37	60.72	58.68	119.40	60.18	59.39	119.57
36	59.37	57.23	116.60	59.45	57.07	116.52
35	57.57	55.61	113.18	57.61	55.69	113.30
34	56.09	54.14	110.23	55.99	54.36	110.35
33	54.48	51.09	105.57	54.48	51.38	105.86
32	51.40	51.49	102.89	51.40	51.52	102.92
31	51.85	49.43	101.28	51.80	49.57	101.37
30	49.79	48.43	98.22	49.40	48.76	98.16
29	48.68	44.83	93.51	48.35	45.22	93.57
28	.....	45.26	.....	45.57	45.08	90.65
27	45.35	43.28	88.63	45.31	43.52	88.83
26	43.15	41.12	84.27	43.63	40.68	84.31
25	40.92	40.22	81.14	41.02	40.24	81.26
24	40.10	37.46	77.56	40.28	37.36	77.74
23	37.68	37.36	75.04	37.67	37.27	74.94
22	.....	33.09	.....	.....	.....	.....
21	.....	.....	69.02	.....	.....	68.82
20	33.29	30.88	64.17	33.03	31.05	64.08
19	30.81	31.45	62.26	30.98	31.50	62.48
18	31.62	28.50	60.12	31.32	28.52	59.84
17	28.72	27.83	56.55	28.62	27.92	56.54
16	27.69	25.29	52.98	27.90	25.23	53.13
15	25.35	23.12	48.47	25.47	23.02	48.49
14	22.99	23.31	46.30	23.16	23.24	46.40
13	23.30	20.03	43.33	23.20	19.83	43.03
12	20.18	19.25	39.43	19.43	19.91	39.34
11	19.43	16.39	35.82	19.60	16.52	36.12
10	16.82	15.74	32.56	16.49	15.91	32.40
9	16.03	14.02	30.05	15.86	14.52	30.38
8	13.84	12.54	26.38	14.40	11.98	26.38
7	12.65	13.40	26.05	12.17	13.45	25.62
6	.....	10.75	.....	.....	10.58	.....
5	.....	.....	17.54	9.40	7.60	17.00
4	.....	7.27	.....	.....	.....	.....
3	6.76?	4.58?	11.34	.....	.....	11.10
2	.....	.....	.....	.....	.....	.....
1	.....	.....	4.81	.....	.....	4.78

cient intensity to appear in any plate, from each one of the branches in the band at $\nu_0=16378.8$, and for the Q and R branches in the band at 17439.5. For this band the intensities of P (22) and P (24)

are very small, the line at R (28) is absent, and lines at Q (29), P (30), and R (32) are abnormally weak.[1] Unfortunately, the difficulty of the observations made it impossible to obtain plates which could be measured with the microphotometer, and all intensities are eye estimates except those in the head of the band, where measurements with a Hartmann microphotometer have been made to determine the presence of lines near the initial points of the bands. But of the reality of the variations of intensity in the alternate weak lines there can be no doubt, since these variations stand out clearly in all the plates and constituted the principal stumbling-block in early attempts to analyze the fine structure of the bands.

The analysis of the structure is further complicated by the presence of bands whose heads lie slightly to the violet of the heads of two of the stronger bands. These fainter bands are weak, so that no extensive analysis can be made. Fortunately, there are enough details so that the sequence of the stronger lines in the branches is clear, and the regions where there is likely to be confusion with the stronger bands can be picked out. One of these overlapping bands, whose head is at $\nu = 15426.6$ and which consequently overlaps the band at $\nu_0 = 15337.6$, shows enough detail on high-dispersion photographs to encourage attempts at analysis, but for some reason these attempts have not been successful. It was thought at first that this and similar overlapping bands might really be part of the stronger bands, indicating incompleteness in the present analysis. There are three reasons why this possibility has been abandoned. First, any analysis including these fainter heads in the structure of the stronger bands failed to take account of all fainter lines in the stronger bands. Second, any supposition that these faint overlapping bands are really slowly turning R branches is contradicted by the rapid degradation of the bands toward the red, since the combination principle fails. Third, the lines in the faint overlapping band whose head is at $\nu = 15426.6$ are much sharper than the lines of the band $\nu_0 = 15337.6$, which it overlaps. The widths of the stronger lines in $\nu = 15426.6$ are between 0.31 and 0.37 in units of wave-number, while the widths of the stronger lines in the band $\nu_0 = 15337.6$ are between 0.82 and

[1] Anomalous intensities of this character are not unknown; an individual case is reported by Mulliken in the *CO* Angstrom bands (*Physical Review*, **29**, 412, 1927).

1.60. The lines of the faint band are sharp, while those of the strong band which it overlaps are extremely diffuse, so much so that it is difficult or impossible to distinguish the alternate weak members

TABLE IV

STRONG LINES ONLY

Band at $\nu_0 = 15337.6$

j	R BRANCH				Q BRANCH				P BRANCH			
	Int.	ν(R)	$\Delta\nu$	$\Delta'\nu$	Int.	ν(Q)	$\Delta\nu$	$\Delta'\nu$	Int.	ν(P)	$\Delta\nu$	$\Delta'\nu$
40	....	.....			2	15037.2			....	.....		
39	<1	15113.0			....	.....	28.2		2	14989.3		
38	....	.....	27.1		3	15065.4		1.1	....	.....	30.8	
37	<1	15140.1		4.8	....	.....	27.1		3	15020.1		2.3
36	....	.....	22.3		3	15092.5		1.4	....	.....	28.5	
35	1	15162.4		0.1	....	.....	25.7		3	15048.6		−1.7
34	....	.....	22.2		3	15118.2		0.8	....	.....	30.2	
33	1	15184.6		1.8	....	.....	24.9		3	15078.8		4.9
32	....	.....	20.4		4	15143.1		2.2	....	.....	25.3	
31	1	15205.0		0.8	....	.....	22.7		4	15104.1		−1.4
30	....	.....	19.6		4	15165.8		0.9	....	.....	26.7	
29	1	15224.6		1.6	....	.....	21.8		4	15130.8		3.9
28	....	.....	18.0		5	15187.6		2.5	....	.....	22.8	
27	2	15242.6		3.2	....	.....	19.3		4	15153.6		−0.2
26	....	.....	14.8		6	15206.9		0.8	....	.....	23.0	
25	2	15257.4		−0.3	....	.....	18.5		5	15176.6		2.1
24	....	.....	15.1		8	15225.4		1.3	....	.....	20.9	
23	1	15272.5		0.9	....	.....	17.2		6	15197.5		1.2
22	....	.....	14.2		11	15242.6		1.2	....	.....	19.7	
21	2	15286.7		2.3	....	.....	16.0		7	15217.2		1.3
20	....	.....	11.9		18	15258.6		1.8	....	.....	18.4	
19	2	15298.6		1.6	....	.....	14.2		8	15235.6		2.0
18	....	.....	10.3		26	15272.8		1.5	....	.....	16.4	
17	2	15308.9		2.3	....	.....	12.7		8	15252.0		0.4
16	....	.....	8.0		24	15285.5		0.3	....	.....	16.0	
15	3	15316.9		1.7	....	.....	12.4		5	15268.0		3.4
14	....	.....	6.3		20	15297.9		2.4	....	.....	12.6	
13	3	15323.2		1.0	....	.....	10.0		5	15280.6		0.8
12	....	.....	5.3		18	15307.9		3.5	....	.....	11.8	
11	2‡	15328.5		−0.9	....	.....	6.5		4	15292.4		−0.6
10	....	.....	6.2		10	15314.4		−1.8	....	.....	12.4	
9	1	15334.7		2.1	....	.....	8.3		6	15304.8		2.8
8	....	.....	4.1		7	15322.7		2.5	....	.....	9.6	
7	1	15338.8†		2.4	....	.....	5.8		3	15314.4		1.3
6	....	.....	1.7		10‡	15328.5		0.8	....	.....	8.3	
5	1	15340.5		1.3	....	.....	5.0		3	15322.7		0.2
4	....	.....	0.4		3	15333.5		3.7?	....	.....	8.1	
3	1	15340.9		1.8	....	.....	1.3		12‡	15330.8		
2	....	.....	− 1.4		5	15334.8			....	.....		
1	1	15339.5			....	.....			*	(15334.7)		

* This line, which should be present, is obscured by R(9) and Q(2).
† This is the middle of an extended hazy patch.
‡ Overlapping faint band obviously has lines at these places.

of the branches. The general character of the lines in the separate bands will be discussed in connection with the analysis of the band system according to the vibrational states.

THE BAND AT $\nu_0 = 15337.6$

While the photographs of this band, like those of $\nu_0 = 16378.8$ and $\nu_0 = 17439.5$, are of good density, they permit a less satisfactory

analysis, because of the extreme fuzziness of the lines. Most of the fainter lines are so blurred as to be indistinguishable from the background of the plate, and an adequate measurement of their wave-lengths is impossible. The determination of the wave-lengths of the R branch is difficult for the reason that the lines nearly all lie close to the red of the strong lines of the Q branch. The analysis of this band is shown in Table IV.

In the case of $\nu = 15242.6$ and $\nu = 15272.8$, the R and Q lines are so close together that they are indistinguishable, the R lines having been shifted slightly to the violet by perturbations. Their existence is indicated by the somewhat greater intensity of the composite line than is normal for members of the Q branch in the vicinity. Attempts to locate the faint alternate lines in this band by means of a recording microphotometer failed because of the extreme widths of the lines, those far out on the branches being too faint and those near the head simply blurring to form a hazy background. The wave-lengths of the strong lines of the three branches, particularly the Q and P branches, can be determined with sufficient accuracy to show the same characteristic perturbations as those of the bands at $\nu_0 = 16378.8$ and $\nu_0 = 17439.5$. This agreement of the perturbations corroborates the close agreement between the $R_j - P_j$ values for the three bands, which identifies them as belonging to the same n''-progression.

As noted before, this band (Table IV) is accompanied by another fainter band whose head is at $\nu = 15426.6$ and which overlaps the head of $\nu_0 = 15337.6$. All attempts to analyze the fainter band have failed. The frequencies and descriptions of the lines are given in Table V. As mentioned previously, the lines are much sharper than those of $\nu_0 = 15337.6$. One reason for the difficulty encountered is obviously that only the region of the head is unobscured by the stronger band. It will be seen from Table V that the arrangements of the stronger lines in what seem to be pairs is entirely missing in this faint band.

OTHER FAINTER BANDS

On small dispersion plates the heads of several other fainter bands can be seen. Of these two have been photographed with high dispersion, but the plates obtained have lacked sufficient intensity

to permit as complete an analysis as the stronger bands. Of these we will discuss first the band at $\nu_0 = 18548.4$. The details of the analysis are shown in Table VI. The frequencies were obtained from two large dispersion plates. It will be seen that only a few of the alter-

TABLE V

ν	Int.	
15426.6	2	about 1ν wide
15424.0	2	
15422.5	1	
15420.5	2	
15419.1	2	
15416.8	2	
15415.4	2	
15412.7	1	
15410.4	2	
15408.8	3	
15406.6	0	very faint
15404.7	4	
15401.5	4	
15397.2	6	possibly close sharp double
15393.4 / 15382.7	5	faint hazy patch on violet side; existence doubtful
15386.3	4	
15383.4	2	hazy patch on red side
15378.0	7	possibly very close sharp double
15373.0	2	continuous haze between these lines
15371.6	1	
15367.1	7	
15363.4	0	existence doubtful
15360.8 / 15358.5		Limits of patch of intensity 1 on violet and 3 on red side
15350.6	6	
15345.1	6	
15340.6 / 15338.0		Hazy patch containing head of band 15338

nate weak lines are measurable, and these, because of their faintness, with scant accuracy. In consequence, the values of the first and second differences, particularly in the R branch, are none too reliable. Hence in Table VI are given the first and second differences of the strong lines only, from which we perceive that this band has in general the same characteristic perturbations in the strong

lines that led to the first steps in the analysis of the three stronger bands. Therefore it should be a member of the same n''-progression,

TABLE VI

BAND AT $\nu_0 = 18548.4$

j	R BRANCH				Q BRANCH				P BRANCH			
	Int.	ν(R)	$\Delta\nu$	$\Delta'\nu$	Int.	ν(Q)	$\Delta\nu$	$\Delta'\nu$	Int.	ν(P)	$\Delta\nu$	$\Delta'\nu$
32	....	.....			1	18338.66			....	.....		
							15.25					
31	<1	18403.45			<1	18353.91			1	18302.56		
							9.88					
30	....	.....			1	18363.79			....	.....		
29	1	18423.72			....	.....			1	18330.08?		
			10.33									
28	<1	18434.05		0.80	1	18387.20			....	.....		
			9.53				11.05					
27	1	18443.58			<1	18398.25		0.20	1	18354.99		
							10.85					
26	....	.....			2	18409.10		0.30	....	.....		
							10.55					
25	<1	18461.19?			1	18419.65		1.08	2	18379.09		
							9.47				11.59	
24	....	.....			3	18429.12		−0.59	1	18390.68		−1.18
							10.06				12.77	
23	1	18478.62			1	18439.18		1.55	1	18403.45?		3.12
			6.93				8.51				9.65	
22	1	18485.55		0.43	4	18447.69			1	18413.10		−0.02
			6.50								9.67	
21	2	18492.05			....	.....			1	18422.77		
20	....	.....			5	18464.93			....	.....		
							7.68					
19	1	18504.07			1	18472.61		−0.14	1	18442.04		
							7.82				8.99	
18	....	.....			5	18480.43		0.65	−1	18451.03		
							7.17					
17	1	18516.99			1	18487.60		0.73	....	.....		
			5.21				6.44					
16	1	18522.20		1.01	6	18494.04			1	18469.41		
			4.20								8.05	
15	2	18526.40		0.57					1	18477.46		1.49
			3.63								6.56	
14	1	18530.03		−0.11	6	18506.53			2	18484.02		−0.31
			3.74								6.87	
13	2	18533.77		−0.94	....	.....			−1	18490.89		
			4.68									
12	<1	18538.45		1.79	5	18516.99			....	.....		
			2.89									
11	1	18541.34		−0.20					2	18505.33		
			3.09									
10	<1	18544.43		0.92	4	18526.40			....	.....		
			2.17									
9	1	18546.60			....	.....			1	18515.72		
8	....	.....			2	18533.77			....	.....		
7	1	18550.00			....	.....			1	18526.40		
6	....	.....			2	18539.89			....	.....		
5	1	18552.31			....	.....			1	18534.90		
4	....	.....			1	18544.43			....	.....		
3	1	18552.50			....	.....			1	18540.80		
2	....	.....			1	18546.60			....	.....		

although we must look to the identity of the combination relation $R_j - P_j$ before putting this as a definite assertion. This detail will be treated later.

The second of the faint bands to yield good results is that at $\nu_0 = 17075.9$. Here the partial success of the analysis is due chiefly to the fact that it occurs in a region easily photographed on a panchromatic plate. In addition its lines are sharp. The results are shown in Table VII. It will be seen that the combination $R_j - Q_j = Q_{j+1} - P_{j+1}$ is none too closely followed. Here, as in the former band, the failure is probably to be attributed to the extreme faintness of the alternate weak lines. Consequently, Table VII has been prepared showing the first and second differences of the strong lines only. While this shows that the R and P branches possess the same perturbation in frequency as all the other bands analyzed, this is not true of the Q branch, which is the strongest and has the most reliable measures.

THE CHARACTER OF THE BAND LINES

A marked feature of these bands is the increase of diffuseness and width of the lines in each band with increasing wave-length. For instance, the lines of the 0,2 band are very sharp, those of the 0,3 band less so, those of the 0,4 band somewhat diffuse, and those of the 0,5 band so diffuse as to make the fainter lines merge into the background of the plate. It was at first thought that this might be due to unresolved fine structure in the bands of longer wave-length. On account of the limit imposed on exposure time by the peculiar difficulties attending the observations, it was not possible to obtain plates in the second order. Visual observations indicate that the appearance of the diffuse lines is not modified by operating the discharge tube in liquid air. Attempts to modify their appearance by changing the pressure in the tube were unsuccessful because it is necessary to maintain a rich flow of fluorine from the generator to obtain a spectrum of observable intensity. It is scarcely possible that the diffuseness is due to an unresolved isotope effect, since Aston[1] has found with the mass spectrograph no evidence of isotopes for fluorine. Furthermore, the observations on the bands indicate that the diffuseness is practically the same for all lines in any branch, in contradiction to the usual isotope effect in bands.

[1] *Proceedings of the Royal Society*, A, **115**, 503, 1927. The exactness of the atomic weight (19) as a whole number makes the existence of isotopes improbable.

TABLE VII

BAND AT 17075.9

j	R BRANCH				Q BRANCH				P BRANCH			
	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$	Int.	ν	$\Delta\nu$	$\Delta'\nu$
40	.....	.....			1?	16752.0*			....	.....		
39	.....	.....			....	.....			....	.....		
38	.....	.....			1?	16782.7*			....	.....		
37	.....	.....			....	.....			....	.....		
36	.....	.....			1?	16811.8*			....	.....		
35	.....	.....			....	.....			1?	16770.6		
34	.....	.....			1	16839.5*			....	.....		
33	.....	.....			....	.....			1?	16800.6		
32	.....	.....			1	16866.5*			....	.....		
31	.....	.....			....	.....			1?	16828.1		
30	.....	.....			2	16890.5*			....	.....		
29	1	16950.0*			....	.....			1	16856.3		
			10.3									
28	<1	16960.3*		0.6	2	16914.6			....	.....		
			9.7				10.2					
27	1	16970.0*		1.4	1?	16924.8		0.3	2	16881.1		
			8.3				9.9					
26	<1	16978.3*		−0.3	2	16934.7			....	.....		
			8.6									
25	2	16986.9*		−0.4	....	.....			2	16905.9		
			9.0									
24	1	16995.9*		1.4	4	16955.7			....	.....		
			7.6									
23	2	17003.5*		0.3	....	.....			3	16928.5		
			7.3									
22	<1	17010.8*		−0.1	5	16974.5			....	.....		
			7.4									
21	3	17018.2*		0.5	....	.....			4	16949.6		
			6.9								11.0	
20	<1?	17025.1*		0.0	6	16991.4			1	16960.6		1.8
			6.9				9.3				9.2	
19	2	17032.0*		1.7	1	17000.7		2.8	4	16969.8		0.7
			5.2				6.5				8.5	
18	<1	17037.2*		−1.4	6	17007.2		−1.2	1	16978.3		−0.1
			6.6				7.7				8.6	
17	2	17043.8*		1.3	1	17014.9		1.8	4	16986.9		−0.4
			5.3				5.9				9.0	
16	1	17049.1*		1.6	8	17020.8		−0.8	1	16995.9		1.5
			3.7				6.7				7.5	
15	3	17052.8			1	17027.5		0.0	4	17003.4		0.1
							6.7				7.4	
14	.....	.....			8	17034.2		2.0	1	17010.8		0.2
							4.7				7.2	
13	4	17062.2			2	17038.9		−1.0	3	17018.0		0.1
			3.6				5.7				7.1	
12	<1	17065.8		0.9	8	17044.6		1.3	1	17025.1		0.1
			2.7				4.5				6.9	
11	3	17068.5			1	17049.1		−1.1	3	17032.0		1.7
							5.6				5.2	
10	.....	.....			7	17054.7		2.8	1?	17037.2		−0.6
							2.8				5.8	
9	2	17074.8			1	17057.5		−1.6	2	17043.0		−0.3
							4.4				6.1	
8	.....	.....			4	17061.9		1.5	<1	17049.1		2.4
							2.9				3.7	
7	1	17077.7			1	17064.8		−0.1	1	17052.8		
							3.0					
6	.....	.....			2	17067.8		0.5	....	.....		
							2.5					
5	1	17079.8			<1	17070.3		0.4	1	17062.2		
							2.1					
4	.....	.....			1	17072.4			....	.....		
3	1	17079.8			....	.....			1	17068.6		
2	.....	.....			1	17074.8			....	.....		
1	.....	.....			....	.....			....	.....		

* These are measurements from a plate of small dispersion—about 16 A per millimeter—the corresponding intensities being reduced to the same scale as those of the large dispersion plates.

It is most probable that the diffuseness is due to the instability of the F_2-molecule under the conditions of excitation. Further evidence of instability is to be found in the fact that even on the densest exposures no lines are found for any branch beyond, approximately, $j=42$.

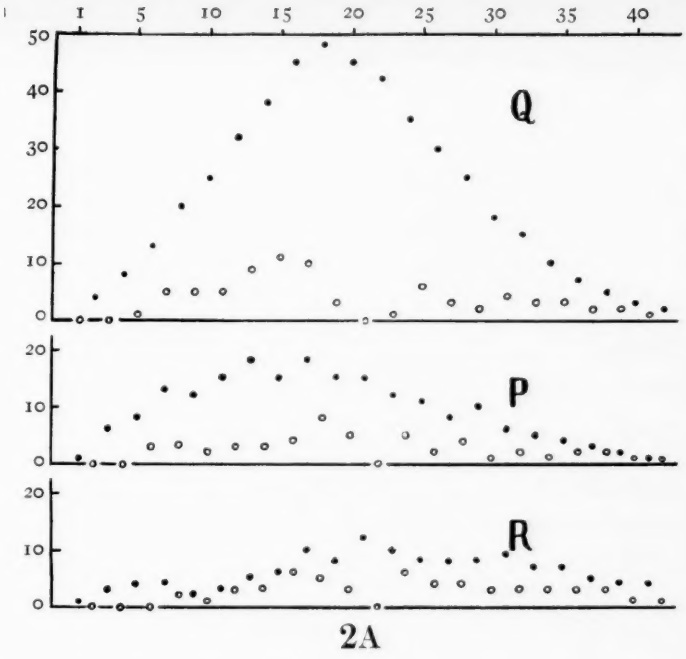
INTENSITIES OF THE P, Q, AND R BRANCHES

Figure 2*A* shows the intensities of the branches for the band at $\nu_0 = 16378.8$; Figure 2*B*, for the band at $\nu_0 = 17439.5$. The intensities are eye measurements, and in the case of small quantum numbers, for lines near the heads of the bands, overlapping lines of the three branches have been assigned intensities which total the observed intensities of the lines in question. But these somewhat arbitrary assignments do not make doubtful the obvious relations between the intensities of the three branches in each band, partly because not all, but only occasional, intensities are thus assigned, and partly because the doubtful cases occur principally in the alternate weak series in each branch.

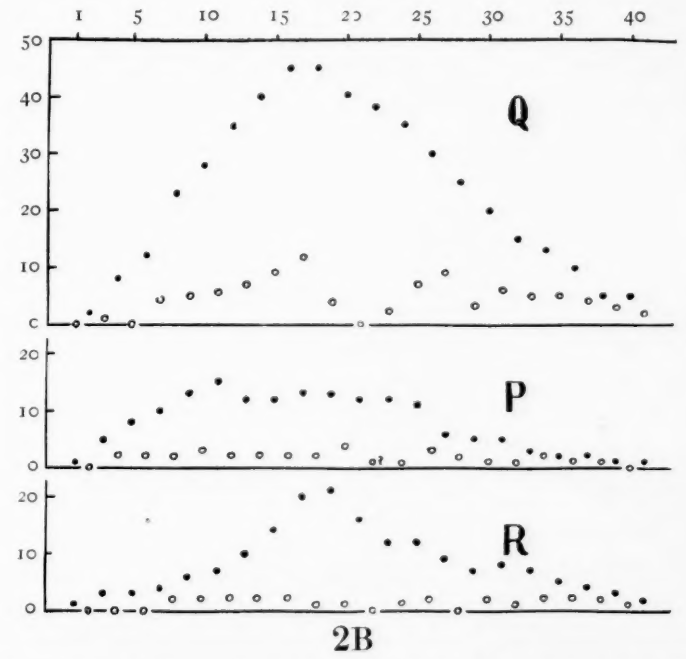
The intensities thus shown, together with the fact that only three branches are found in each band, point definitely to a band system due to a $^1S \rightarrow {}^1P$ transition. Figure 3 shows the transitions involved. P(1), Q(2), and R(1) should be present, while P(2), Q(1), and R(2) should be present, but very faint. The observations shown in Tables I and II are in agreement with this requirement, P(2), Q(1), and R(2) obviously not being observed because of their small intensity.

The intensity measurements shown in Figure 2, although based on eye estimates, are believed to be sufficiently accurate for an estimate of the relationship between the alternate strong and weak lines. Hund[1] states that a ratio of intensities of 2:1 indicates a nuclear spin of 2 units, i.e., $h/2\pi$ erg. sec., and that a ratio of 3:1 indicates a nuclear spin of 1 unit, i.e., $\frac{1}{2}(h/2\pi)$. Figure 2 shows the ratio to be certainly greater than 2:1 and somewhere between 3:1 and 4:1. The difference in the relative intensities of strong and weak lines for the R branches for the two bands seems to be real, although it might be due to an effect on the estimated intensities of the relatively greater diffuseness of the 0,4 band lines.

[1] F. Hund, *Zeitschrift für Physik*, **42**, 112, 1927; see also W. Heisenberg,, *ibid.*, **41**, 239, 1927, and D. M. Dennison, *Nature*, **119**, 316, 1927.



2A



2B

FIG. 2.—Fig. 2*A* shows estimated intensities of lines of band at 16378.8; Fig. 2*B*, of band at 17439.5. The abscissae are values of j. The ordinates are intensities. Zero intensity means that the line is absent; intensity 1, that it is just seen. The black dots show the intensities of the strong lines of each branch; the circles, the weak alternate lines.

The greater estimated intensities for many of the strong lines in the R branch of the band at 17439.5, as compared with corresponding lines in the band at 16378.8, are apparently real. It is possible that the difference is due to the greater diffuseness of the lines in the latter band.

DETERMINATION OF THE VIBRATIONAL STATES

While the frequencies of all the band heads have been classified according to a definite system of vibrational states, the character of the fine structure of the two completely analyzed bands and the three partly analyzed bands has been examined for necessary

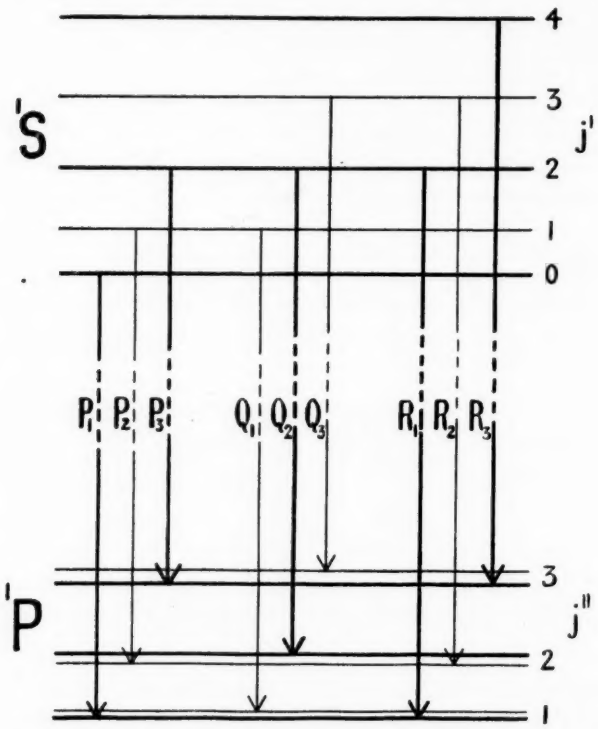


FIG. 3.—Schematic arrangement of energy-levels for a $^1S \rightarrow {}^1P$ band system with alternate strong and weak lines. The first lines of each branch are shown.

corroboratory evidence concerning this classification. Table VIII shows a tabulation of the $R_j - P_j$ combinations for four of the bands. In this table, while the close agreement of columns 3 and 4 places the two strong bands definitely in the same n''-progression, the evidence for the other two is not so good. But it must be remembered that measurements on the fainter lines for these bands are almost entirely missing; further, that the band at 18548.4 is extremely faint, data from several plates being patched together, and that the

band at $\nu_0 = 15337.6$ is not only overlaid by another band but has very diffuse lines. In view of these facts, the data have been considered good enough to warrant placing, tentatively, all the bands in the same n''-progression.

TABLE VIII

COMPARISONS OF COMBINATION $R_j - P_j$ FOR FOUR BANDS

1	2	3	4	5
j	18548.4	17439.5	16378.8	15337.6
42	.....	.....	132.64	.....
41	.....	131.87	131.86	.....
40	.....	.....	129.29	.....
39	.....	125.41	125.54	123.7?
38	.....	122.60	122.44	.....
37	.....	119.40	119.57	120.0
36	.....	116.60	116.52	.....
35	.....	113.18	113.30	113.8
34	.....	110.23	110.35	.....
33	.....	105.57	105.86	105.8
32	.....	102.89	102.92	.....
31	100.89	101.28	101.37	100.9
30	.....	98.22	98.16	.....
29	93.64	93.51	93.57	93.8
28	.....	.....	90.65	.....
27	88.59	88.63	88.83	89.0
26	.....	84.27	84.31	.....
25	82.10?	81.14	81.26	80.8
24	.....	77.56	77.74	.....
23	75.17	75.04	74.94	75.0
22	72.45	.....	.....	.....
21	69.28	69.02	68.82	69.5
20	.....	64.17	64.08	.....
19	62.03	62.26	62.48	63.0
18	.....	60.12	59.84	.....
17	.....	56.55	56.54	56.9
16	52.79?	52.98	53.13	.....
15	48.94	48.47	48.49	48.9
14	46.01	46.30	46.40	.....
13	42.88	43.33	43.03	42.6
12	.....	39.43	39.34	.....
11	36.01	35.82	36.12	36.1
10	.....	32.56	32.40	.....
9	30.88	30.05	30.38	29.9
8	.....	26.38	26.38	.....
7	23.60?	26.05	25.62	24.4?
6	.....	.....	.....	.....
5	17.41	17.54	17.00	17.8
4	.....	.....	.....	.....
3	11.70	11.34?	11.10	10.1?
2	.....	.....	.....	.....
1	.....	4.81	4.78	4.8

Table IX gives the combinations of $R_{j-1}-P_{j+1}$ for five bands, four of which are also shown in Table VIII. As before, the figures

TABLE IX

COMPARISON OF $R_{j-1}-P_{j+1}=\Delta_2F''(j)$ FOR FIVE BANDS

j	15337.6	16378.8	17439.5	18548.4	17075.9
41	.....	164.46	.....	.....	.....
40	.....	159.74	161.97	.....	.....
39	.....	155.96	.....	.....	.....
38	150.8	152.05	153.68	.....	.....
37	.....	148.25	150.38	.....	.....
36	142.3	144.29	145.98	.....	.....
35	.....	140.29	142.25	.....	.....
34	136.0	136.63	138.18	.....	.....
33	.....	132.60	133.97	.....	.....
32	126.2	128.79	130.10	.....	.....
31	.....	124.58	126.56	.....	.....
30	120.5	121.06	122.46	121.16	.....
29	.....	117.82?	.....	.....	.....
28	111.8	113.19	114.42	113.50	113.7
27	.....	109.10	110.52	.....	.....
26	103.8	105.54	106.46	106.20	105.8
25	.....	101.40	102.54	.....	.....
24	95.9	97.34	98.65	99.53	97.6
23	.....	.....	.....	.....	.....
22	89.2	89.22	90.90	88.60	89.7
21	.....	.....	84.22?	.....	.....
20	81.4	81.69	82.45	81.30	82.4
19	.....	77.66	78.78	.....	76.6
18	73.3	74.20	74.80	74.95	74.0
17	.....	69.27	70.50	71.17	70.8
16	64.9	65.86	66.42	.....	65.9
15	.....	61.48	62.10	60.62	.....
14	55.2	57.61	58.49	56.31	58.8
13	.....	53.78	54.36	54.43	55.0
12	47.9	48.79	50.61	50.45	50.5
11	.....	44.14	45.87	.....	.....
10	42.3	41.80	40.53	41.27	42.8
9	.....	38.20	37.14	.....	.....
8	34.0	33.70	34.25	34.28	34.7
7	.....	.....	.....	.....	.....
6	26.1	27.42	27.42	25.91	27.0
5	.....	.....	.....	.....	.....
4	18.2	17.30	18.75	17.60	17.6
3	.....	.....	.....	.....	.....
2	.....	10.40	10.56	.....	.....
1	.....	.....	.....	.....	.....

for bands at 15337.6, 18548.4, and 17075.9 are only fragmentary. One might say that nothing definite can be decided from this table concerning the assignment of 17075.9 to a given vibrational state. There is, however, a tendency of the values for that band to cor-

respond to those of 16378.8.[1] If this indicates anything, it means that the two bands belong to the same n'-progression, the fainter band having a larger n'-value. It will be found that these assignments are corroborated by the classification in Table X.

The constants of the equation in Table X were obtained by a least-squares reduction of the observations, with suitable weights assigned to the best bands. For the 0,0 band (for which the n''-value is more or less arbitrary), the 0,1; 0,2; and 0,3 bands there is a wide spread between the calculated and observed values. A slight difference is of course due to the fact that for all except the five bands analyzed only the head has been measured, but in no case could this difference exceed two or three in wave-number. The principal reason for the poor fit for the four bands mentioned above is a marked perturbation of the 0,2 band to the violet. The 0,0 band was at first observed visually only with very low dispersion, as it is extremely faint. Later, close scrutiny of some plates taken with a Hilger E_3 quartz spectrograph showed a hazy patch at the approximate position of the visual observation. All the other bands appear on an exposure made with a concave grating of 2-m radius, although the 0,1; 2,6; and 2,7 bands are extremely faint. It is difficult to understand why the 0,2 band should be perturbed, except that in a system of bands containing so many perturbations in intensity and frequency in the fine structure, some perturbations in the vibrational states are not unexpected, especially since such perturbations have been observed in other band spectra. The partial analysis of the band in question makes the position of its initial point certain, within the limits of experimental error. In Table X the intensities of the observed bands are indicated by Roman numerals in brackets.

It should be pointed out that the labeling of the band at 20738.0 as the 0,0 band is arbitrary, although the n' sequence is probably correct. The resulting values of the constants, ω_0', ω_0'', $\omega_0'x'$, and $\omega_0''x''$ are, of course, dependent on the assumption as to the 0,0 band. In order to fit all the observed bands into a scheme of vibrational states, it is necessary to assign a value to $\omega_0'x'$ of 140.6. This

[1] The Δ_2F_j'' values for the band at $\nu_0 = 18548.4$ also agree with those for $\nu_0 = 16378.8$ and $\nu_0 = 17075.9$; but since there is a perturbation in the vibrational state, an abnormality in the spacing of the rotational levels is also reasonable.

TABLE X

The Vibrational States*

$$\nu = \nu_0 + (836.83n' - 140.6n'^2) - (1130.1n'' - 9.7n''^2)$$

$n''=$	0	Δ	1	Δ	2	Δ	3	Δ	4	Δ	5	Δ	6	Δ	7
	(VII)		(IV)		(III)		(I)		(I)		(II)		(IV)		
$n'=0$	Cal. 20754.4	*1129.4*	19625.0	*1101.1*	18523.9	*1081.8*	17442.1	*1062.4*	16379.7	*1043.1*	15336.6	*1022.6*	14314.0	.....	.....
	Obs. **20738.0**	*1108.0*	**19630.0**	*1081.6*	**18548.4**	*1108.9*	**17439.5**	*1060.7*	**16378.8**	*1041.2*	**15337.6**	*1022.6*	**14315.0**	.....	.....
Δ=	.....	.....	.....	.....	.....	.....	*696.2*	.....	*696.2*	.....	.....	.....	.....	.....	.....
	.....	.....	.....	.....	.....	.....	*696.5*	.....	*697.1*	.....	.....	.....	.....	.....	.....
							(IV)		(V)						
$n'=1$	.....	.....	.....	.....	19220.1	.....	18138.3	*1062.4*	17075.9	.....	16032.8	.....	15110.2	.....	.....
	.....	.....	.....	.....	.....	.....	**18138.0**	*1062.1*	**17075.9**	.....	.....	.....	.....	.....	.....
Δ=	.....	.....	.....	.....	.....	.....	.....	.....	*415.0*	.....	.....	.....	.....	.....	.....
	.....	.....	.....	.....	.....	.....	.....	.....	*416.1*	.....	.....	.....	.....	.....	.....
									(IV)				(III)		(IV)
$n'=2$	.....	.....	.....	.....	.....	.....	18553.3	.....	17490.9	.....	16447.8	.....	15425.2	*1005.6*	14419.6
	.....	.....	.....	.....	.....	.....	.....	.....	**17492.0**	.....	.....	.....	**15425.0**	*1008.0*	14417.0

* Bold-face type indicates observations; italics indicate differences.

is somewhat large, but a similar case is found in the $F-XCO$ bands, in which $\omega_0' x'$ has a value of 198. In the HgH bands, $\omega_0'' x'' = 104.0$.

THE SOLUTIONS FOR B_0' AND B_0''

The data for the separate bands, as given in Tables VIII and IX, can be substituted in the expression $\Delta_2 F(j) = 4B(j+\frac{1}{2})$, where j is the number in column 1, to obtain a rough average value of B, by means of the formula

$$B = \frac{\Sigma \Delta_2 F(j)}{2\Sigma(2j+1)}.$$

Table XI gives the values thus found for B':

TABLE XI

n'	n''	Band	Weight	B_0'
0.....	2	18548.4	1	0.821
0.....	3	17439.5	6	.814
0.....	4	16378.8	5	.813
0.....	5	15337.6	2	0.817

An examination of Tables VIII and IX shows that the values of $F''(j)$ are much more uniform than those of $F'(j)$, which is what was

TABLE XII

n'	n''	Band	Weight	B''
0.....	2	18548.4	1	1.025
0.....	3	17439.5	6	1.025
0.....	4	16378.8	5	1.011
0.....	5	15337.6	2	1.043

to be expected from the fact that the perturbations are found in the initial state. Consequently, the B''-values for the strongest bands should yield a better value of α and B_0 in the expression $B_n = B_0 - \alpha n$. Since there are only two such bands, however, we cannot expect much from Table XII.

However, Birge[1] has found the following relation from which α

[1] *Physical Review*, **31**, 919, 1928. Paper presented to the Berkeley meeting of the American Physical Society, March 3, 1928.

may be determined to the correct order of magnitude:

$$R=\frac{2x}{a/B_0},$$

where R has the value 1.4 ± 0.2 and x is obtained from the expression $\omega_n=\omega_0-2x\omega_0 n$. Applying this test to the data for the best band, we get $x''=0.0084$ and $a''=0.013$. This agrees well with the value of $a''=0.014$ obtained from Table XII for the 0,3 and 0,4 bands; the other values are so widely divergent that nothing can be obtained from them. If the value of a'' is put at 0.014, the value of B_0'' is 1.073, using the weights given in Tables XI and XII. The ratio of B_0' to B_0'' is then found to be 0.76, as compared to the ratio ω_0' to ω_0'', which is 0.74.

From the foregoing values for B_0' and B_0'' we get the values of the moments of inertia for the initial and final states of the F_2 molecule, and the internuclear distances

$$I_0'=34.2\times10^{-40},$$
$$I_0''=26.0\times10^{-40},$$
$$r_0'=1.48\times10^{-8}\text{ cm}.$$
$$r_0''=1.28\times10^{-8}\text{ cm},$$

We take pleasure in acknowledging our indebtedness to Professors R. S. Mulliken and R. T. Birge for many helpful suggestions regarding the analysis of these bands.

RYERSON PHYSICAL LABORATORY
UNIVERSITY OF CHICAGO
January 10, 1929

A SPIRAL NEBULA AS A STELLAR SYSTEM, MESSIER 31[1]

By EDWIN HUBBLE

ABSTRACT

Material.—The present discussion of M 31 is *based on the study of about 350 photographs* taken with the 60- and 100-inch reflectors, distributed over an interval of about eighteen years. Two-thirds of the total number were obtained by the writer during the five years 1923–1928. Since the image of the nebula is much larger than the usable fields of the telescopes, *attention was concentrated on four regions centered* on (1) the nucleus, (2) 23′ north following, (3) 17′ south, (4) 48′ south preceding the nucleus. The combined *area*, with allowance for overlapping, *represents about 40 per cent of the entire nebula.*

Resolution.—The *outer regions* of the spiral arms are *partially resolved into swarms of faint stars*, while the *nuclear* region *shows no* indications of *resolution under any conditions* with the 100-inch reflector. Intermediate regions show isolated patches where resolution is pronounced or suggested.

Variables.—Fifty variables have been found, nearly all in the outer regions where resolution is pronounced. The *survey* is believed to be *fairly exhaustive in the four selected regions down to 19.0 photographic magnitude.*

Cepheids.—Forty of the *variables* are *known to be Cepheids* with periods from *48 days to 10 days* and *maxima from 18.1 to 19.3 photographic magnitude;* one exceptional star varies from 17.9 to 19.2 in a period of 175 days. The *period-luminosity relation is conspicuous*, and the slope is *approximately that* found among *Cepheids in other extra-galactic systems.*

Distance of M 31 derived from Cepheid criteria.—Comparisons of period-luminosity diagrams indicate that M 31 is about 0.1 mag. or 5 per cent more distant than M 33, and about 8.5 times more distant than the Small Magellanic Cloud. Using Shapley's value for the Cloud, we find the *distance of M 31* to be *275,000 parsecs.*

Variables other than Cepheids.—Of the 10 remaining variables, 4 are probably very faint Cepheids for which the data are insufficient to establish the characteristics, and 6 are irregular or long-period variables. The latter group includes the brightest variables in the nebula.

Novae.—Sixty-three novae have been found by the writer, which, together with the 22 previously observed, gives a *total of 85 observed photographically* which are now *available* for *statistical investigation.* The novae exhibit a striking similarity in their behavior, and the *mean light-curve is of the same general character as that for galactic novae.* The frequency distribution of magnitudes at maxima can be represented by an error-curve with a maximum at about 16.5 ($M = -5.7$ for the distance indicated by the Cepheids) and a probable error for a single nova of about 0.5 mag. Selective effects appear to be unimportant, and a restricted range of about 4 mag. seems to be established.

Novae are most frequent in the nuclear region, and, in a general way, the distribution follows that of the luminosity in the nebula. Position in the nebula appears to have no effect on the luminosity of novae. It is *estimated* that *novae appear at the rate of about 30 per year.*

Brighter non-variable stars.—Preliminary surveys suggest that few stars in M 31 are brighter than $M = -6$ (distance, that indicated by the Cepheids) and that large numbers are to be found only for M fainter than -5. A study of the open cluster N.G.C. 206, involved in M 31, indicates that the average color-index of the brighter stars is of the order +0.3 or +0.4 mag.

Mass and luminosity densities in M 31.—The mass density of M 31 appears to be of the order of one sun per 20 cubic parsecs, and the luminosity density, about 9.0 mag. per

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 376.

cubic parsec. With the sun as unit, $Mass = 5.5L$. These values are of the *same order as those for the galactic system* in the neighborhood of the sun. As the nucleus is approached, the coefficient of the mass-luminosity relation decreases until, for the nucleus itself (diameter about 4 parsecs), it falls below 0.001.

Relative dimensions of M 31 and the galactic system.—A tentative comparison of sizes, masses, luminosities, and densities suggests that the *galactic system is much larger than M 31* but that the ratio is not greater than that between M 31 and other known extra-galactic systems.

Messier 31, the great nebula in Andromeda, is the most conspicuous of all the spirals and the only one which can be seen easily with the naked eye. It appears as a hazy patch about 30′ by 15′, somewhat brighter in the center, with a total luminosity variously estimated as from the fourth to the fifth magnitude. The object is listed as a nebula in the tenth-century star catalogue of Al-Sufi, and appears on some of the pre-telescopic star charts. It was first examined with a telescope in 1612 by Marius, who gave the famous description, "like a candle seen through a horn." Of the many subsequent drawings and descriptions that were based on visual observations, the most notable are those by Bond and by Trouvelot.

In 1885 interest in the nebula was stimulated by the appearance of a nova very close to the nucleus, which reached the eighth magnitude. Two years later the photographs made by Isaac Roberts with his 20-inch reflector showed for the first time the main pattern of the spiral structure. Ranyard reproduced one of Roberts' photographs in *Knowledge* for February, 1889, and the article which accompanied it was the best discussion of a spiral nebula which had appeared up to that date.[1] Among other things he mentioned the numerous stars in the outer regions of the nebula, the significance of which was not fully appreciated until many years later.

Early visual observers of the spectrum reported bright lines on a continuous background. In 1899, however, Scheiner[2] photographed the now familiar solar-type absorption spectrum and announced emphatically that the nebula must be a system of stars. Radial velocities of the order of -300 km/sec. have since been measured by several observers. The inclination of the lines which appears when the slit is oriented along the major axis, first reported by Slipher, was

[1] This first reproduction was from a drawing copied from the negative. The same volume, however, contains two photographic reproductions, the later of which, in the August number, is exceptionally good.

[2] *Astrophysical Journal*, **9**, 149, 1899.

measured by Pease[1] in 1917. The linear velocity of rotation as indicated by the measures is of the order of $0.48x$ km/sec., where x is the distance from the nucleus in seconds of arc. The measures extend to about 150″ from the nucleus, and the rotation is in the sense that the south preceding end of the nebula is approaching us relative to the nucleus.

Ritchey,[2] in 1917, discovered two faint novae on his earlier photographs of M 31, and since then the observers at Mount Wilson, following the nebula for the purpose, have found eighty-three others. In the course of this work the writer, in 1923, found two faint variable stars which further investigation proved to be Cepheids with periods of twenty and thirty-one days, respectively. About the same time, photographs centered along the outer arms of the spiral revealed a high degree of resolution into individual stars (Plate III), which was entirely lacking in the nuclear region where previous plates had been centered. This opened a new field for investigation—the study of individual stars involved in a spiral nebula—which has been developed primarily by means of the 100-inch reflector.

Thus far some fifty variables have been found in M 31, and forty of them are known to be Cepheids. The period-luminosity relation is conspicuous among the Cepheids, hence the distance of the spiral is determined with considerable accuracy in terms of the distance of the Magellanic Clouds. Shapley's value for the zero point of the period-luminosity relation leads to a provisional value for the absolute distance.

The stars in M 31, including the novae and the Cepheids, are so closely comparable with those in the neighboring spiral, M 33, that the two systems may be combined for a general discussion. The considerable numerical data resulting from this combination, together with the consistency of the various independent criteria, confirm the general order of the distances derived from the Cepheids and suggest no revision of Shapley's value for the zero point of the period-luminosity relation. The data for M 33 have already been published,[3] and will be used freely in the present discussion of M 31.

[1] *Proceedings of the National Academy of Sciences*, **4**, 21, 1918.

[2] *Publications of the Astronomical Society of the Pacific*, **29**, 210, 1917.

[3] *Mt. Wilson Contr.*, No. 310; *Astrophysical Journal*, **63**, 236, 1926.

VARIABLE STARS

M 31 is an intermediate-type spiral, Sb in the writer's classification,[1] with its equatorial plane inclined about 15° to the line of sight. The unresolved nuclear region is about 30′×10′, and the maximum extension of the arms, on long exposures, is about 160′×40′. The arms exhibit considerable resolution, and there are patches, especially in the south preceding end, where the stars fairly swarm on the photographic plates.

Fifty variables have been found among the stars scattered over the image of the spiral, and most of these undoubtedly belong to the nebula. The search is believed to be fairly exhaustive, down to 19.0 photographic magnitude, in four regions of the nebula, centered as follows:

Region 1. The nucleus
2. 23′ north following the nucleus
3. 17′ south of the nucleus
4. 48′ south preceding the nucleus

The usable fields around these centers have radii of from 12′ to 15′, and the combined area, with allowance for overlap in Regions 1–3, represents about 40 per cent of the entire nebula.

For the central region and for the brighter stars in the adjacent regions as well, some two hundred and seventy plates covering a period of eighteen years are available. A series of ten photographs by Ritchey in the autumn of 1909 is followed by a gap of eight years during which only three usable plates were obtained, but from 1917 on the material is fairly well distributed. Previous to 1920 the plates were all obtained with the 60-inch reflector; since then the 60- and the 100-inch have been used indiscriminately. The exposures and seeing conditions vary widely since many of the plates were obtained in the search for the relatively bright novae. The better photographs represent average exposures of about sixty minutes with the 60-inch reflector and thirty to forty minutes with the 100-inch, although occasional plates were exposed up to five hours or more. In general, fast plates were used throughout.

The photographs centered on Regions 2, 3, and 4 represent exposures of seventy-five to ninety minutes with the 100-inch reflec-

[1] *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926.

tor and, in general, are of a rather better quality. The variables in Regions 2 and 3 have been followed consistently only since 1924, but some of them are found on the larger and longer-exposed plates of the earlier series. The season 1924–1925 is especially well represented. Two series of fifteen and ten consecutive nights, respectively, during which the observing conditions were uniformly excellent, indicated immediately the general nature of the light-curves of many of the variables, and suggested periods which were confirmed by reference to earlier plates. Variables Nos. 5 and 10, with about sixty-five observations each, are the least well observed.

Some seventy-five plates of Region 4 have been assembled, representing exposures on about sixty different nights. The material appears to be rather scanty, but, in this region as well as in the others, each variable for which a period has been determined has been followed for at least one complete season after the determination, and in no case has this test led to a radical revision. Periods close to an even day, or submultiple of a day, have been definitely eliminated by sets of plates obtained on single nights. In all, about twenty such sets were made, often on successive nights, representing intervals ranging from two to nine hours.

Sequences of comparison stars with steps averaging about 0.2 mag. were selected for each variable and were calibrated by some sixteen comparisons with Selected Areas 20, 21, 44, and 45. The magnitudes are all photographic. The unpublished magnitudes of stars in the Selected Areas, available through the kindness of Mr. Seares, extend to about 18.5. Fainter magnitudes are extrapolations; but to 19.0 they are believed to be of the same accuracy as the fainter stars of the Selected Areas. From 19.0 to 20.0 the extrapolations must be considered as increasingly unreliable, although the results from the various plates are in substantial agreement. In the case of a few variables, notably Nos. 3 and 4, the background of nebulosity is so dense that the measures are very uncertain even for the brighter stars.

The magnitudes of the variables, estimated to the nearest tenth by comparison with the sequences, are believed to be fairly trustworthy (except where the plates were exposed under poor conditions) over the brighter portion of the light-curves. The minima, however,

TABLE I

VARIABLES IN M 31, CENTRAL REGION

J.D.*	Plate†	Qy.	1‖	2	3	4	5§	6	7	8	9
8562.7‡	R18	F	19.0	19.1+	18.6	19.3+	18.8+	.....	18.9	19.5	.....
8563.5‡	20	G	19.15	19.2	18.85	19.7	18.9+	18.7	18.9	19.6	.....
8565.5	26	G	19.15	18.6	18.9	19.7	19.1+	18.9	19.0	19.7	19.3
8566.6	27	F	19.0+	18.6	18.9	19.4+	18.8+	18.7	18.9	19.7	.....
8591.5	34	G	19.05	19.3	18.9	19.6	18.9	18.6	18.4	19.5	19.4
8593.4	39	G	19.1	19.2	19.3	19.1	18.7	18.9	18.4	19.2	19.4
8885.6	108	G	19.3	19.0	18.4	19.1	.....	19.5	18.95	19.7	.....
0720.5	121	E	18.35	19.1	18.4	19.1	.....	19.0	18.85	20.0	.....
0781.5	122	E	18.25	19.1	19.3	19.0	.....	19.6	18.2	19.3	.....
1462.6	P300	F	19.2+	18.8	19.2+	19.1	.....	.....	18.2	19.5	.....
1483.5	Sh3991	P	18.7+	19.0+	.....	.....	.....	19.0	18.7	.....	.....
1489.5	R146	G	19.3	19.1+	19.2+	19.4+	.....	19.4+	18.8	19.0	.....
1518.5	147	F	19.3	18.6	.....	.....	.....	.....	18.7	.....	.....
1547.5	151	G	19.2	19.1+	19.3+	19.4	.....	18.7	18.1	19.5	.....
1575.4	154	G	19.1	19.1+	19.2+	19.5+	.....	19.5	19.0	19.5	.....
1576.4	156	G	19.2	19.15	19.4	19.5+	.....	19.5	18.8	19.3	.....
1609.3	161	G	19.3	19.4	19.4	19.2	.....	18.9	18.7	19.6	.....
1634.3	162	P	18.8+	19.1+	19.2+	19.3+	.....	18.6	18.5	19.4	.....
1635.3	163	P	18.8+	.....	19.2+	.....	.....	18.7	18.4	19.4	.....
1781.6	D2	P	18.6+	18.9	19.1	.....	.....	.....	18.3	.....	.....
1782.6	5	P	18.2	18.8	.....	.....	.....	.....	.....	.....	.....
1783.6	9	P	18.2	18.9	.....	.....	.....	.....	.....	.....	.....
1811.3‡	11	P	19.0+	19.0+	.....	19.0	.....	18.65	18.85	19.3	.....
2104.7	Sh4934	P	18.6	18.8	.....	.....	.....	.....	.....	.....	.....
2161.6	4978	F	18.25	18.7	.....	19.1	.....	.....	18.8	.....	.....
2163.6	5025	F	18.6	18.5	.....	19.3	.....	.....	18.8	19.0	.....
2168.6	D24	P	18.85	19.2+	.....	.....	.....	18.7	.....	.....	.....
2169.6	27	F	19.0	19.1	19.4+	19.5+	.....	18.8	18.9	19.5+	.....
2195.5	30	F	18.6	19.1+	19.3+	19.5+	.....	18.8	18.6	19.3+	.....
2223.5	Sh5054	P	18.3	18.7	.....	19.0	.....	.....	.....	.....	.....
2263.4	5117	G	19.0	18.8	19.1	19.3	.....	.....	18.7	19.6	18.9
2311.3	5149	F	19.0+	19.2	.....	.....	.....	.....	18.4	19.4	.....
2314.3	5160	G	19.0	19.2+	18.4	.....	.....	19.5	18.1	19.6	19.2+
2317.3	Hg2	G	18.25	19.3+	.....	.....	.....	.....	18.2	.....	18.2
2521.6	D37	P	18.8+	19.2+	19.2+	19.4+	.....	18.6	19.0	19.2	.....
2545.6	Sh5489	F	18.9	19.0	.....	.....	.....	18.6	18.2	.....	18.2
2547.6‡	D42	F	19.1	19.2	19.4+	19.7	18.55	18.6	18.3	.....	18.4
2548.6	44	E	19.15	19.2	19.7	19.7	18.5	18.7	18.3	19.8	18.4
2580.5	62	F	19.15	19.1+	19.3+	19.4	.....	19.6	18.6	19.5	.....
2581.6	65	P	.....	18.8+	19.2+	.....	.....	.....	18.4	19.4+	.....
2584.4	70	F	19.2	18.7	18.4	19.5+	19.0+	19.6	18.1	.....	18.8
2585.0‡	71	G	19.4	18.7	18.4	19.6+	19.3+	19.6	18.1	19.7	18.8
2585.3	72	P	19.2	18.8	.....	.....	19.0+	.....	18.1	.....	18.9
2608.5	76	F	18.9	19.2	19.2+	19.4+	.....	19.3+	18.9	19.0	.....
2609.6	80	P	18.8+	18.9+	19.4	.....	.....	18.7	18.9	19.2	.....
2612.3	S68	F	19.1	19.1+	18.3	19.3+	.....	18.7	18.8	19.3	.....
2641.6	75	F	19.1	19.2+	19.1	.....	.....	.....	18.6	19.2	.....
2642.5	77	G	19.1	19.0	18.6	19.6	.....	.....	.....	19.1	.....
2669.3‡	D106	F	18.8	18.9+	18.9	19.4+	.....	.....	18.6	19.5	.....
2694.3‡	118	G	18.2	19.2+	18.65	19.5+	.....	.....	18.6	19.4	.....
2695.3	122	F	18.3	19.1+	18.9	19.5	.....	.....	18.9	19.1	.....
2698.4‡	131	F	18.5	19.1+	19.1	19.2	.....	19.2+	18.85	19.05	.....
2903.4‡	200	P	18.8+	18.8+	.....	19.0	.....	18.8	18.2	.....	.....
2935.5	H150	E	.....	.....	.....	.....	18.8	.....	.....	.....	.....
3029.4	A10	F	19.3	19.1	19.5	19.2	.....	.....	18.8	19.4+	.....
3256.6	29	P	19.1	19.1+	19.4	19.2	.....	18.7	18.7	19.6+	.....
3258.7	B51	F	18.2	19.0+	.....	.....	.....	18.8	.....	.....	.....
3259.6	56	P	18.2	19.0+	18.6	.....	.....	18.8	.....	.....	.....
3260.7	S156	P	18.25	18.7+	18.4	.....	.....	.....	18.1	.....	.....
3264.6	D222	P	18.6	19.0+	.....	.....	.....	.....	18.3	19.0	.....

TABLE I

Variables in M 31, Central Region

J.D.	10§	11	12	13	14	15	16	17	18	19
8562.7‡	.....	.....	.....	.....	.....	17.5	.....	.....	.....	16.0
8563.5‡	19 0+	.....	.....	.....	.....	17.5	.....	19.35	.....	16.0
8565.5	19.3	19.5+	19.5+	19.3	.....	17.5	.....	19.0	.....	16.0
8566.6	.....	.....	19.3+	19.3	.....	17.5	19.5+	18.8	19.7+	16.0
8591.5	19.2+	19.5	19.5	19.5	19.8	17.6	19.3	19.9	19.9	16.0
8593.4	19.2+	.....	.....	19.8	.....	17.6	19.3	19.9	19.2	15.9
8885.6	.....	19.5+	19.1	19.7+	.....	17.5	19.3	19.1	19.7+	15.85
0720.5	.....	19.5	19.8+	19.5	19.7	17.4	18.8	19.7	19.9+	15.3
0781.5	.....	19.3	.....	19.1	18.9	17.3	18.5	18.9	19.2	15.3
1462.6	.....	18.9	19.0	.....	.....	17.0	18.8	.....	19.5+	15.8
1483.5	.....	18.6	.....	.....	.....	17.1	.....	.....	.....	15.6
1489.5	.....	18.5	19.2	.....	.....	17.0	.....	.....	.....	15.6
1518.5	.....	18.7	19.4+	19.4	19.6+	17.0	19.4	19.5	19.6+	15.5
1547.5	.....	18.6	19.4+	19.5	.....	16.9	.....	.....	19.6+	15.55
1575.4	.....	18.6	19.15	19.6+	19.1	16.9	.....	.....	19.6+	15.6
1576.4	.....	18.7	19.4	19.6+	19.0	16.9	.....	19.9	19.6+	15.6
1609.3	.....	19.0	.....	19.4	19.3	16.9	19.5+	19.1	19.6+	15.7
1634.3	.....	19.2	.....	.....	.....	16.8	19.1	.....	19.3	15.7
1635.3	.....	19.1	.....	.....	.....	17.0	18.9	.....	19.2	15.8
1781.6	.....	.....	.....	.....	.....	.....	.....	.....	.....	16.15
1782.6	.....	.....	.....	.....	.....	17.1	.....	.....	.....	16.2
1783.6	.....	.....	.....	.....	.....	16.9	.....	.....	.....	16.1
1811.3‡	.....	19.2	.....	.....	.....	17.0	.....	.....	.....	16.15
2104.7	.....	.....	.....	.....	.....	17.1	.....	.....	.....	16.2
2161.6	.....	.....	.....	19.5+	19.5	17.2	18.9	.....	.....	16.3
2163.6	.....	.....	.....	.....	.....	17.1	18.8	19.6+	.....	16.3
2168.6	.....	.....	.....	.....	.....	16.8	.....	.....	.....	16.2
2169.6	.....	19.3	.....	.....	.....	.....	19.4	18.9	19.6	16.2
2195.5	.....	19.2	19.1	19.6+	19.3	16.9	19.4+	.....	19.5+	16.3
2223.5	.....	.....	.....	.....	.....	17.1	.....	.....	.....	16.4
2263.4	.....	19.4	19.1	19.6+	19.0	17.0	19.4	18.9	19.3	16.2
2311.3	.....	19.5	.....	19.5+	.....	17.1	.....	.....	.....	16.2
2314.3	.....	19.3	19.4+	19.5+	19.2	16.8	19.5+	19.6+	19.5+	16.2
2317.3	.....	.....	19.4+	19.6+	19.3	.....	.....	.....	.....	16.1
2521.6	.....	19.2	.....	19.5	.....	16.8	.....	.....	.....	15.6
2545.6	.....	.....	.....	.....	18.9	16.8	.....	.....	19.5	15.7
2547.6‡	18.8	19.3+	19.5+	19.7	19.3	16.9	19.45	19.15	19.7	15.7
2548.6	18.9	19.3	19.5	19.7	19.4	16.9	19.3	19.0	19.7	15.6
2580.5	.....	.....	.....	.....	18.9	.....	19.3	19.3	19.3	15.5
2581.6	.....	.....	.....	.....	.....	17.0	.....	19.1	.....	15.5
2584.4	19.2+	19.4+	19.5+	19.2	19.5	16.9	19.5	19.1	19.5	.....
2585.0‡	19.4	19.3	19.8+	19.2	19.5	17.0	19.3	19.1	19.8	.....
2585.3	19.2+	.....	.....	.....	.....	.....	.....	.....	.....	.....
2608.5	.....	.....	.....	19.4	.....	.....	19.3+	.....	.....	15.5
2609.6	.....	.....	.....	.....	.....	16.9	.....	.....	.....	15.4
2612.3	.....	19.4	.....	.....	.....	17.1	18.9	.....	.....	15.4
2641.6	.....	.....	.....	.....	.....	17.0	.....	.....	.....	15.4
2642.5	.....	.....	.....	.....	19.4+	17.1	.....	.....	19.6+	15.3
2669.3‡	.....	.....	.....	.....	18.9	17.0	.....	.....	.....	15.4
2694.3‡	.....	19.5+	19.3	19.5+	19.4+	16.8	19.1	19.1	19.4	15.4
2695.3	.....	.....	.....	19.5+	19.4+	17.0	18.8	18.75	19.5	15.5
2698.4‡	.....	.....	19.2+	.....	.....	16.9	18.8	.....	19.6	15.6
2903.4‡	.....	.....	.....	.....	.....	16.8	.....	.....	.....	15.3
2935.5	18.8	.....	.....	.....	.....	.....	19.8+	19.8	19.7	15.3
3029.4	.....	19.2+	.....	.....	19.6	.....	18.9	.....	19.5	15.3
3256.6	.....	.....	.....	.....	19.6	.....	.....	.....	.....	15.9
3258.7	.....	.....	.....	.....	.....	.....	.....	18.7	.....	16.0
3259.6	.....	.....	.....	.....	.....	.....	.....	18.75	.....	15.9
3260.7	.....	.....	.....	.....	.....	16.8	.....	.....	.....	15.85
3264.6	.....	.....	.....	.....	.....	16.8	.....	.....	.....	15.9

TABLE I—*Continued*

J.D.*	Plate†	Qy.	1‖	2	3	4	5§	6	7	8	9
3287.6	B58	P	18.6+	18.6	18.6	.....	.....	.....	.....	19.2	.....
3289.6	62	P	18.25	18.9+	18.6	.....	.....	.....	18.9	.....	.....
3317.7	S172	F	18.9+	19.1+	18.7	.....	.....	.....	18.4	18.9	.....
3350.3	A42	P	18.8+	18.7	.....	.....	.....	.....	.....	.....	.....
3433.3	B100	F	19.1	19.1	19.5	.....	.....	18.7	18.8	19.5+	.....
3610.6	126	F	18.5	19.0	18.6	.....	.....	.....	18.9	19.3	.....
3612.6	131	P	19.0	19.2	.....	.....	.....	.....	19.0	.....	.....
3615.7	A58	P	19.15	19.1+	.....	.....	.....	.....	18.8	.....	.....
3617.7	S254	P	19.1	18.7+	18.9	.....	.....	.....	18.5	19.4+	.....
3639.7	267	F	18.3	18.8+	.....	.....	.....	19.1	18.6	18.9	.....
3641.5	S278	P	18.5	18.8+	18.5	.....	.....	.....	.....	19.2+	.....
3698.6	H335	G	18.3	19.4	19.4	19.7	.....	.....	.....	19.7	.....
3737.6	339	G	18.85	19.2	19.5	19.7	.....	19.6+	18.9	19.6	.....
3765.45	348	F	18.5	19.2+	19.5	19.7	.....	19.5	18.3	19.2	.....
3768.4	351	P	18.7	18.55	.....	.....	.....	18.7	18.5	19.5	.....
3789.3	353	P	18.8+	18.6	.....	.....	.....	18.6	.....	19.3+	
3790.4	S285	F	18.8	18.8	19.4+	19.1	.....	18.6	.....	19.5+	18.8
3791.4	292	F	18.3	18.8	19.3+	19.0	.....	18.5	.....	19.6	18.9
3818.3	301	F	19.3	19.1+	19.0	19.3+	.....	.....	18.55	19.2	18.4
3819.3	310	F	19.1+	19.1+	19.1+	19.3+	.....	.....	18.65	19.3	18.5
3820.3	320	F	19.1+	19.3	.....	.....	.....	.....	18.7	.....	18.6
3821.3	H355	G	19.1	19.1+	19.5	19.7	.....	19.3	18.65	19.5	.....
3822.3	357	F	18.6	19.2+	19.6+	19.6	.....	19.5	18.7	19.7	.....
3823.3	363	F	18.3	19.2+	19.6+	19.6+	.....	19.5+	18.7	19.6	.....
3843.3	A73	P	.....	.....	.....	.....	.....	.....	.....	.....	.....
3969.6‡	H376	P	.....	18.7	18.8	.....	18.9	.....	.....	.....	.....
3970.6	381	F	.....	.....	.....	19.3+	.....	18.9	19.2	19.3+	19.4+
3971.6	S329	P	19.2+	18.5	.....	.....	.....	.....	.....	.....	.....
3990.6‡	336	P	.....	18.6	18.4	.....	.....	.....	.....	.....	.....
3991.6	338	G	19.3	18.85	18.7	.....	19.4+	18.6	.....	19.3	.....
3993.6	S339	G	19.3	18.95	18.9	.....	19.4+	18.6	.....	19.1	.....
3994.6‡	343,4	F	19.4	19.0	19.1	19.3+	19.6	18.9	18.6	19.0	19.4
3995.6‡	348,9	G	19.4	19.2	18.9	19.2	19.5	18.8	18.6	19.15	19.3
3996.6‡	H384,5	E	19.4	19.4	19.3	19.0	19.3	18.9	18.7	19.3	19.4
3997.6‡	389,0	G	19.4	19.4	19.5	19.0	19.1	19.0	18.5	19.4+	19.4
3998.6‡	394,5	E	19.4	19.4	19.5	19.2	18.9	19.3	18.5	19.3+	19.4
3999.6‡	S350,1	G	19.4	19.4	19.5	.....	18.6	19.1	18.7	.....	19.4+
4000.6‡	H397	F	19.4	19.3	19.6	.....	18.7	.....	18.7	19.3+	19.4
4001.6‡	398	F	19.3	19.5	19.5	.....	18.6	.....	18.8	19.3+	19.4
4002.6‡	403,4	E	19.4	19.3+	19.55	19.4+	18.55	19.5	18.9	19.4+	.....
4003.6‡	409,0	E	19.4	19.5	.....	.....	18.5	19.6	18.8	19.3+	19.4
4004.5‡	414,5	E	19.4	19.5	19.5	.....	18.6	.....	19.0	19.5+	19.5
4023.4	B182	F	19.3	19.4+	19.4	.....	19.3+	.....	.....	.....	.....
4024.5‡	D238,1	F	19.2	19.4+	19.25	19.3+	19.3	19.5	18.2	19.7	19.4
4025.5‡	243,6	F	19.4	19.4	19.3	19.7	19.3	19.5	18.1	19.5	19.3
4026.5‡	S353,6	F	19.3	19.5	19.3+	.....	19.3	19.5	18.2	19.4+	19.3
4027.6‡	359,0	F	19.4	19.35	19.5	.....	19.2+	19.7	18.2	18.9	19.4
4028.5	H418	G	.....	19.1	19.5+	19.6+	.....	19.8	18.1	19.0	.....
4029.4	S361	F	19.1+	18.75	.....	.....	19.1+	.....	.....	.....	.....
4030.5	362	F	.....	18.6	.....	.....	.....	19.7	18.25	19.1	19.4
4031.5‡	H420,1	G	19.1+	18.6	19.4	.....	19.4	19.5	18.2	19.3	19.4
4032.5‡	424,6	G	19.3	18.9	19.6	19.7	19.2+	18.6	18.35	19.3	19.4+
4033.6‡	427,0	G	19.3+	19.05	19.7+	19.3+	19.3	18.75	18.3	19.5	19.4
4049.5‡	S366,0	F	18.8	18.6	18.7	.....	18.6	19.4+	18.8	.....	18.75
4050.5‡	372,4	F	18.9	18.55	19.1	.....	18.6	19.7	18.8	19.4+	18.8
4054.6‡	H434	F	.....	.....	.....	19.4+	.....	.....	18.9	19.4+	18.9
4055.6	S376	G	19.2	19.15	19.3+	.....	18.6	18.7	.....	.....	.....
4056.6	385	G	.....	19.2	.....	.....	18.75	18.6	19.0	19.5+	19.1
4057.4	388	G	19.3	19.3	19.7	19.0	18.7	18.6	19.1	19.6+	19.3
4059.4‡	H443,4	G	.....	.....	.....	19.0	18.7	18.6	19.0	19.5+	19.3

TABLE I—*Continued*

J.D.	10§	11	12	13	14	15	16	17	18	19
3287.6	.....	.....	.....	.....	.....	.....	.....	.....	.....	16.0
3289.6	.....	.....	19.3	.....	.....	16.7	.....	.....	.....	16.0
3317.7	.....	19.1	.....	.....	.....	16.8	.....	18.75	19.4+	16.2
3350.3	.....	.....	19.1	.....	.....	.....	.....	.....	.....	16.0
3433.3	.....	.....	.....	.....	.....	.....	19.3	19.4	.....	16.2
3610.6	.....	.....	.....	.....	.....	16.9	19.2	19.4+	.....	16.0
3612.6	.....	.....	19.1	.....	.....	.....	19.3	.....	.....	16.1
3615.7	.....	.....	.....	.....	.....	.....	.....	18.85	.....	.....
3617.7	.....	.....	.....	.....	.....	16.8	.....	.....	.....	16.0
3639.7	.....	.....	.....	.....	19.3	16.8	18.9	.....	.....	16.2
3641.5	.....	.....	.....	.....	.....	16.9	.....	.....	.....	16.1
3698.6	.....	19.4	.....	.....	19.5+	.....	.....	.....	.....	16.2
3737.6	.....	19.3	.....	19.5	19.5+	16.9	.....	.....	19.6+	16.3
3765.45	.....	19.1	19.3+	19.5	19.6	17.0	18.85	.....	19.1	16.3
3768.4	.....	.....	.....	.....	.....	.....	.....	.....	.....	16.2
3789.3	.....	.....	.....	.....	.....	17.0	19.5	.....	19.6	16.1
3790.4	.....	19.1	.....	19.5	19.5+	17.0	19.5+	19.6	19.6	16.2
3791.4	.....	19.1	19.4+	19.6+	19.6+	17.1	.....	.....	19.8	16.1
3818.3	.....	.....	19.0	19.3+	19.4+	17.2	19.2	.....	19.3	16.2
3819.3	.....	19.3+	19.3	19.4+	19.6	17.2	19.3	19.6+	19.4+	16.2
3820.3	.....	.....	19.1	19.3+	19.3+	17.1	19.4	19.3	19.25	16.3
3821.3	.....	19.4+	19.2	19.2	19.6+	17.0	.....	.....	19.2	16.3
3822.3	.....	19.3+	19.2+	19.2	19.5+	17.1	.....	.....	19.3	16.3
3823.3	.....	.....	19.3+	19.2+	19.4+	17.0	.....	.....	19.3	16.3
3843.3	.....	.....	.....	.....	.....	.....	.....	18.8	.....	16.3
3969.6‡	19.1+	.....	.....	.....	.....	.....	18.85	19.4	19.0	16.1
3970.6	.....	.....	19.6+	19.7+	19.7+	16.9	.....	.....	.....	16.2
3971.6	.....	.....	.....	.....	.....	.....	19.1	.....	.....	.....
3990.6‡	.....	.....	.....	.....	.....	16.7	.....	18.7	.....	16.3
3991.6	18.9	19.4	.....	.....	.....	16.8	19.5	18.75	19.4	16.3
3993.6	19.0	19.3	.....	.....	.....	.....	19.5	19.1	19.6	16.2
3994.6‡	19.2	19.3	19.2	19.7	19.4	17.0	19.7	19.25	19.7	16.3
3995.6‡	19.2	.....	19.2	19.2	19.5	16.8	19.8	19.3	19.7	16.3
3996.6‡	19.2	.....	19.0	19.2	19.8	16.8	19.8	19.35	19.7+	16.2
3997.6‡	19.3	.....	19.2	19.3	19.8	16.8	19.6	19.6	19.9	16.2
3998.6‡	19.4	.....	19.2	19.4	19.8+	16.9	19.8	19.6	19.9	16.1
3999.6‡	19.4+	19.3	19.35	19.4	19.8+	17.0	19.7	19.7	19.8+	16.0
4000.6‡	.....	.....	19.4	19.4	19.8+	17.0	19.9	19.7	19.9	16.2
4001.6‡	19.3	.....	19.4	19.4	19.8+	16.8	19.8+	19.7	19.9	16.1
4002.6‡	.....	19.1	19.4	19.5	19.8+	16.7	19.7+	19.7	19.8+	16.2
4003.6‡	19.5	19.3	19.6	19.6	19.9	16.7	19.8	19.7	19.9	16.2
4004.5‡	19.4	.....	19.6	19.6	.....	16.9	19.9	19.7	.....	16.2
4023.4	19.4+	.....	.....	.....	19.6+	.....	19.15	19.8+	19.7+	.....
4024.5‡	19.4+	19.2	19.7	19.7	19.8	16.6	19.2	19.9	19.2	16.0
4025.5‡	19.5+	19.3	19.7	19.8	19.2	16.7	19.35	19.9	19.2	16.1
4026.5‡	19.4+	19.1	19.7	19.8	19.0	16.8	19.4	19.4	19.2	16.2
4027.6‡	19.2	19.1	19.7	19.8	19.05	16.8	19.4	18.75	19.3	16.1
4028.5	.....	19.3	19.7+	19.8+	19.2	16.7	.....	.....	.....	15.9
4029.4	18.8	.....	.....	19.4+	19.2	16.8	19.5	19.1	19.5+	16.0
4030.5	.....	19.1	19.8+	19.8+	19.3	16.9	.....	.....	.....	16.1
4031.5‡	18.9	19.1	19.8	19.8+	19.7	16.7	19.4	19.1	19.7+	16.1
4032.5‡	18.95	.....	19.7+	19.8+	19.7	16.8	19.5	19.4	19.7+	16.1
4033.6‡	.....	19.15	19.7+	19.8+	19.9	16.7	19.6	19.35	19.7	16.2
4049.5‡	19.3+	19.0	19.8+	19.8+	.....	16.7	19.2	19.0	19.7+	16.3
4050.5‡	19.1	18.9	19.8+	19.9	19.8	16.7	19.1	18.9	19.7	16.3
4054.6‡	.....	.....	19.8	19.8	19.8	16.8	.....	.....	.....	16.2
4055.6	18.8	19.0	.....	.....	19.8	.....	18.85	19.4	19.7+	16.2
4056.6	18.95	18.9	19.8	19.9	19.9+	16.9	.....	.....	.....	16.2
4057.4	18.95	19.0	19.8	20.0	20.0	16.7	18.8	19.6	.....	16.2
4059.4‡	19.0	18.8	19.7	19.8	19.8+	16.7	18.9	19.7	19.7+	16.2

TABLE I—*Continued*

J.D.*	Plate†	Qy.	1‖	2	3	4	5§	6	7	8	9
4085.4	451	G	.....	.....	19.4	.....	19.2+	.....	.....	.....	.....
4086.4‡	S394,5	F	19.1	19.3+	19.3+	.....	19.1+	.....	18.5	.....	19.05
4089.4‡	400,1	G	19.2	18.8	19.4+	.....	19.5+	19.15	18.7	19.6	19.3
4090.4	H457	G	.....	.....	.....	.....	.....	.....	18.7	19.7	19.4
4092.3	464	F	.....	18.6	.....	.....	.....	19.5	18.8	.....	19.4
4108.4‡	H469,0	F	18.45	19.2	19.5+	.....	18.85	.....	18.8	19.5+	18.2
4109.3	471	F	.....	.....	.....	19.7	.....	19.5	18.8	19.7+	18.3
4114.4	S411	G	19.0	19.0	19.4	19.7	19.05	19.6	18.2	19.4+	18.7
4150.3	H510	P	19.1+	18.55	18.4	.....	.....	19.3+	18.9+	.....	.....
4168.3	S417	P	18.2	18.8+	.....	.....	.....	18.6	18.4	.....	19.2+
4177.4	424	F	19.0	19.2+	18.7	.....	.....	19.1	18.6	19.6	.....
4178.3	433	F	19.1	19.2+	18.4	19.5+	.....	.....	18.8	19.6+	18.8
4181.3	H531	P	19.1	19.4	18.7	19.0	.....	.....	18.9	.....	.....
4197.3	S434	P	19.1	19.4	.....	.....	.....	.....	19.2	.....	19.2+
4207.3	446	P	.....	.....	.....	.....	.....	.....	18.15	.....	.....
4319.6	H571	F	19.4	19.3	19.3	19.7	.....	19.6	18.7	19.6+	.....
4321.6	S483	G	19.4	19.3+	19.1	.....	19.2+	18.8	.....	.....	.....
4349.5	H580	F	.....	19.1	19.3+	19.0	.....	19.0	18.4	19.1	.....
4351.6	D251	F	19.4	18.5	19.5	19.2	.....	19.3	18.6	19.2	19.1
4356.6	268	G	18.2	19.2	19.5	19.5	.....	19.5	18.7	19.6+	18.9+
4358.5	S496	G	18.2	19.3	19.4+	19.6	.....	19.7	18.7	19.5+	19.3+
4377.4	H586	F	19.3+	19.3	19.3	19.6+	.....	19.3	18.9	19.7+	.....
4379.7	598	P	19.2+	19.2	19.3+	.....	.....	19.3	18.9	19.7	.....
4406.6	607	P	.....	.....	.....	.....	.....	.....	18.9	19.3+	18.8
4414.7	609	F	.....	18.9	19.5+	19.2	.....	18.6	19.0	19.8	18.9
4433.3	615	P	.....	18.6	19.4	.....	.....	.....	18.0	19.3+	.....
4441.5	S505	F	19.3+	19.2	.....	.....	.....	19.0	18.3	19.5	18.9
4444.4	H617	F	.....	.....	.....	19.6	.....	19.6	18.5	19.7	18.9
4475.3	621	P	.....	.....	18.6	.....	.....	19.3+	18.1	19.5	.....
4476.5	Sr6276	P	19.2+	19.1	18.4	.....	.....	.....	18.2	.....	.....
4493.4	H624	F	19.1	18.85	19.5	19.1	.....	19.6	18.7	19.2	.....
4495.3	626	P	19.3	19.0	.....	.....	18.95	.....	.....	.....	.....
4522.3	627	P	.....	.....	.....	.....	.....	18.6	18.2	.....	.....
4530.3	SS45	P	19.2+	19.3	18.4	19.3+	.....	19.1	18.4	19.5	.....
4531.3	H629	P	19.2+	18.65	18.6	19.3	.....	19.3	18.4	.....	.....
4671.6	S533	P	18.4	19.0	.....	.....	.....	.....	18.65	.....	.....
4680.6	D304	P	19.0	19.2+	.....	.....	.....	18.6	18.9	.....	.....
4700.6	315	G	18.7	19.4	19.5	19.2	.....	18.6	18.0	19.7	.....
4709.6	H668	F	18.9	19.4	19.5	19.6	.....	19.5	18.4	19.5	.....
4711.6	S539	F	19.1	19.3	.....	.....	.....	.....	18.3	.....	.....
4739.5	540	P	18.6	19.15	.....	19.0	.....	19.6	19.0	19.0	.....
4762.4	H675	G	19.3	19.4	19.6	19.2	.....	19.6	18.55	19.5	.....
4764.4‡	686	E	18.25	19.5	19.4	19.3	.....	19.6	18.6	19.6	.....
4770.4	690	E	18.9	19.4	19.1	19.6	.....	18.6	18.8	19.6	.....
4772.4‡	703	E	18.8	18.75	18.4	19.6	.....	18.9	18.9	19.5+	.....
4793.3	S545	F	19.2	18.45	19.5	19.5+	.....	18.6	18.1	19.2	18.6
4800.3	H708	E	18.5	19.2	18.4	19.0	.....	19.6	18.3	19.6	.....
4801.3	S547	P	18.6	19.3	18.7	19.1	.....	.....	18.5	.....	.....
4822.5	BB104	F	19.3+	19.5	.....	19.2	.....	19.6	18.8	.....	.....
4823.5	117	F	19.4	19.3+	19.3+	19.3	.....	.....	18.9	19.5	.....
4882.3	166	F	19.4+	19.4+	18.5	18.9	.....	18.7	18.2	19.0	.....
4883.3	180	P	19.1+	19.1+	18.7	19.1	.....	18.8	18.0	.....	.....
4908.3	H715	G	19.4	19.6	18.4	19.4	.....	19.3	19.0	19.6+	.....
4918.3	720	F	19.2+	19.05	19.3+	19.4+	.....	.....	19.0	19.2	.....
4938.3	732	P	19.2	19.0	18.6	.....	.....	.....	18.5	19.2	.....
5035.6	780	P	19.3	18.5	.....	.....	.....	18.7	18.8	19.6	.....
5036.6	785	F	19.3	18.7	19.6	19.6+	.....	18.8	18.8	19.7	.....
5063.6	798	F	19.1	19.5	19.6+	19.5	.....	19.4	18.2	19.5	.....
5083.5	812	F	18.6	19.5	19.8	19.8	.....	19.2	18.7	19.6	.....
5091.5	815	P	.....	.....	19.3+	19.2	.....	.....	19.0	.....	.....

TABLE I—*Continued*

J.D.	10§	11	12	13	14	15	16	17	18	19
4085.4	19.1+	.....	.....	.....	.....	.....	19.8	18.85	19.4	.....
4086.4‡	19.1+	19.0	19.35	19.4	19.6+	16.7	.....	19.1	19.5	16.2
4089.4‡	19.3+	19.2	19.6	19.7	19.8	16.7	19.4	19.4	19.8	16.1
4090.4	.....	18.9	19.7	19.8	19.6+	.....	.....	.....	.....	16.15
4092.3	.....	19.1	19.6+	19.6+	19.7	16.7	.....	.....	.....	.....
4108.4‡	19.1	19.0	19.1	.....	.....	16.8	19.3	19.2	19.5	16.2
4109.3	.....	19.1	19.2	19.5	19.8	16.9	.....	.....	.....	16.3
4114.4	19.4+	19.0	19.4	19.5	19.1	16.9	19.5	19.4	19.7+	16.1
4150.3	.....	19.2	18.9	.....	19.0	16.9	.....	.....	.....	16.2
4168.3	.....	.....	.....	.....	19.2	.....	.....	.....	.....	16.0
4177.4	.....	.....	19.5	.....	19.6+	16.8	18.85	19.0	19.7+	16.2
4178.3	.....	19.3	.....	.....	19.6+	16.8	18.9	18.75	.....	16.2
4181.3	.....	.....	19.6	.....	.....	16.8	.....	.....	.....	16.3
4197.3	.....	19.2	19.3	.....	.....	.....	19.3	18.8	19.7+	16.2
4207.3	.....	.....	.....	.....	.....	.....	.....	.....	.....	16.1
4319.6	.....	19.0	19.3+	19.6+	19.6+	16.9	.....	.....	19.5	15.9
4321.6	19.3+	19.2	.....	.....	.....	16.8	19.4	19.6	19.0	16.0
4349.5	.....	.....	19.2+	.....	.....	16.7	.....	.....	.....	16.0
4351.6	.....	19.2	18.8	.....	19.7	.....	19.3	19.35	19.8+	16.0
4356.6	.....	19.3	19.2+	.....	.....	16.7	19.5	19.7	19.7	15.9
4358.5	.....	19.4	19.4	19.6+	19.7+	16.8	19.4	19.7+	19.1	15.8
4377.4	.....	19.3+	19.0	19.6+	19.4	16.7	.....	.....	.....	15.9
4379.7	.....	.....	.....	.....	18.9	16.8	.....	.....	.....	15.8
4406.6	.....	.....	.....	.....	.....	16.8	.....	.....	.....	15.7
4414.7	.....	19.5+	19.6+	19.6	19.0	16.7	.....	.....	.....	15.7
4433.3	.....	.....	.....	.....	19.2	16.7	.....	.....	.....	15.7
4441.5	.....	19.4+	18.9	19.5+	19.4+	16.7	19.6	18.7	19.7+	15.6
4444.4	.....	19.4+	19.3	.....	.....	16.8	.....	.....	.....	15.6
4475.3	.....	19.4+	.....	19.0	.....	16.8	.....	.....	.....	15.3
4476.5	.....	.....	.....	19.1	.....	16.8	.....	.....	.....	15.4
4493.4	.....	19.6	.....	.....	.....	16.9	.....	.....	.....	15.3
4495.3	19.2	.....	.....	.....	.....	.....	19.7	19.7	19.7+	15.4
4522.3	.....	.....	.....	.....	.....	.....	.....	.....	.....	15.3
4530.3	.....	.....	.....	.....	.....	17.0	.....	.....	.....	15.4
4531.3	.....	.....	.....	.....	.....	16.9	.....	.....	.....	15.4
4671.6	.....	.....	.....	.....	.....	17.3	.....	.....	.....	15.55
4680.6	.....	.....	.....	.....	19.3	17.3	19.3	19.4+	19.5+	15.55
4700.6	.....	.....	.....	.....	.....	17.2	.....	.....	19.7	15.4
4709.6	.....	.....	.....	.....	.....	17.3	.....	.....	19.3	15.5
4711.6	.....	.....	.....	.....	.....	17.3	18.7	18.9	19.1	15.4
4739.5	.....	.....	.....	19.2	.....	17.2	19.5+	19.2	19.6+	15.5
4762.4	.....	19.6	19.6+	19.6+	19.7	17.2	18.9	.....	19.8+	15.55
4764.4‡	.....	.....	.....	.....	19.7	17.1	.....	.....	19.8+	15.6
4770.4	.....	19.6	19.4	.....	19.4	17.2	.....	.....	.....	15.7
4772.4‡	.....	19.6	.....	.....	19.7	17.1	19.4	19.7	19.7	15.6
4793.3	.....	.....	.....	.....	.....	17.2	18.9	19.7	19.7+	15.6
4800.3	.....	19.6+	19.3	19.0	19.7+	17.3	19.05	.....	.....	15.6
4801.3	.....	.....	19.3+	19.1	19.0	17.3	19.2	18.9	.....	15.7
4822.5	.....	.....	19.4+	18.9	19.1	17.2	19.5+	19.2	19.1	15.6
4823.5	.....	19.4	19.3+	19.2	19.4	17.2	19.5+	19.2	19.2	15.6
4882.3	.....	.....	.....	.....	.....	17.2	19.1	19.4	.....	15.4
4883.3	.....	.....	.....	.....	.....	17.2	18.85	19.3+	.....	15.5
4908.3	.....	19.5+	18.9	19.1	19.2	17.2	.....	.....	.....	15.4
4918.3	.....	.....	19.2+	19.3+	19.4+	17.1	18.75	19.5	19.25	15.4
4938.3	.....	.....	.....	.....	.....	17.2	.....	.....	.....	15.45
5035.6	.....	.....	19.2+	.....	.....	17.2	.....	.....	.....	15.8
5036.6	.....	19.6+	.....	19.8	19.8+	17.1	.....	.....	19.4+	15.9
5063.6	.....	.....	19.1	19.5	19.8+	17.2	19.5	.....	19.15	16.0
5083.5	.....	.....	.....	19.1	19.5	.....	18.8	.....	19.35	16.0
5091.5	.....	.....	.....	.....	19.7	17.1	19.1	.....	19.7	16.0

TABLE I—*Continued*

J.D.*	Plate†	Qy.	1‖	2	3	4	5§	6	7	8	9
5115.4.....	H824	F	18.7	18.6	19.6	19.3	.....	19.6	18.2	19.4	.....
5116.4.....	828	F	18.8	18.8	19.6	19.4	.....	19.7	18.3	19.4	.....
5124.4.....	H832	P	19.2	19.6	18.7	19.7	.....	18.9	18.5	19.7	.....
5145.4.....	S577	F	18.3	19.5+	19.4+	19.5+	.....	18.8	18.9	19.5	.....
5147.6.....	H848	P	18.6	.....	19.2	.....	.....	19.0	18.8	19.1	.....
5149.6.....	H855	F	18.9	19.6+	18.7	19.5	.....	19.1	18.4	19.1	.....

are often below the limits of the plates, and generally below the limits of accuracy. For this reason statistical discussions must be confined largely to the magnitudes at maxima.

Magnitudes for the variables in Regions 1, 2, and 3 are listed in Table I, and for those in Region 4, in Table II. Values for different plates exposed during a single night are generally combined, as they seldom differ by more than the errors of observation.

TABLE I—*Continued*

J.D.	10§	11	12	13	14	15	16	17	18	19
5115.4	.....	19.6	19.3+	19.7	19.8	17.0	.....	.....	.....	16.0
5116.4	.....	19.6	19.4+	19.5+	19.8	17.0	.....	.....	19.4+	16.0
5124.4	.....	19.8	.....	19.4	19.6	17.0	.....	.....	19.4+	16.0
5145.4	.....	.....	.....	.....	.....	17.2	.....	.....	.....	16.0
5147.6	.....	.....	.....	19.0	.....	17.1	.....	.....	.....	16.0
5149.6	.....	19.6	19.1	19.3	.....	17.0	.....	.....	19.7	16.0

* Add 2,410,000 to the first seven dates and 2,420,000 to the remainder.

† Letters preceding the plate numbers refer to the following observers: R = Ritchey, P = Pease, Sh = Shapley, D = Duncan, Hg = Hoge, S and H = Hubble, A and B = Humason, Sr = Seares, SS = Schilt, BB = Brown.

‡ Nights on which more than one plate were obtained.

|| A plus sign following magnitude indicates that the variable was invisible, but certainly fainter than the tabulated value.

§ See Table I*a* for additional observations.

TABLE I*a*

Additional Observations, Variables in M 31, Central Region; Variables Nos. 5 and 10

J.D.	Plate	Qy.	5	10
4054.4	H432	P	.....	18.8
4056.4	S379	G	18.65	.....
4113.4	408	P	18.9	19.2+
4168.3	416	P	19.1+	18.85
4323.6	494	F	19.2+	19.2+
4384.5	D276	E	18.65	19.2+
4387.5	287	E	18.5	19.2+
4731.5	323	F	.....	19.4+
4739.5	S541	F	19.0	18.9

TABLE II

VARIABLES IN M 31, REGION 4

J.D.*	Plate†	Qy.	21‖	22	23	24	25	26	27	28
4032.4	H424	F	19.9	20.1	19.5	19.4	19.7	19.05	19.0	19.1
4054.4	432	P	19.2	19.5+	19.6	19.6	19.8+	19.7	19.4	19.3
4056.4	S379	G	19.0	19.3	18.7	19.8	19.8	19.1	19.3	19.25
4058.55	H438	F	19.2	18.95	19.0	19.8	20.0	19.1	18.7	19.2
4085.6	454	E	19.9	19.7	19.8	19.2	19.2	19.2	19.0	19.4
4090.55	461	P	19.1	19.5	18.7	19.7	19.8	19.6	19.5	19.2
4108.35	468	F	19.1	19.5	18.8	19.7	18.95	19.45	18.8	19.2
4113.4	S408	P	19.35	.....	19.2	.....	.....	19.2	.....	19.3
4115.5	H478	F	19.8	19.4	19.4	19.9	19.9	19.5	19.4	19.4
4117.5	497	F	19.8	19.7	.....	19.8+	19.7	19.45	19.5	19.3
4147.4	504	P	19.5	19.1	19.3	19.7+	19.55	19.7+	19.5	18.3
4151.4	512	F	19.9	19.5	19.7	19.9	19.7+	19.8	19.5	18.8
4168.3	S416	P	.....	.....	.....	.....	.....	.....	.....	.....
4171.3	H516	G	20.0	19.7	19.6	20.0	19.7	19.6	19.5	19.3
4180.3	525	F	19.35	18.9	18.8	19.7	19.4	19.9	18.9	18.7
4198.3	S439	P	.....	.....	.....	.....	.....	.....	.....	.....
4320.6	H573	F	19.8	19.1	18.8	19.8	19.6	19.0	19.5	19.15
4323.6	S494	F	.....	.....	19.2	19.7+	19.5	19.2	18.7	19.1
4349.6	H581	G	19.3	19.7	19.8	19.25	19.9	19.35	18.95	19.2
4353.6	D257	P	19.6	.....	18.8	19.55	19.8+	19.6	18.9	19.2
4357.5	H583	G	20.1	19.0	19.1	19.8	19.05	19.6	19.4	19.3
4377.5	587	E	20.1	19.1	19.4	20.0+	19.7	19.4	19.2	19.3
4379.5	599	P	.....	.....	.....	.....	.....	19.4+	19.3	.....
4384.5	D276	E	19.3	19.8	19.6	19.2	19.7	20.0	19.5	19.35
4387.45	287	E	19.5	19.8	19.6	19.55	20.1	19.9	19.4	18.8
4405.4	H605	G	19.6	19.7	19.45	19.5	19.4	19.5	19.4	19.3
4414.4	610	F	19.6	19.5	19.5	20.0	19.5	19.9	19.5	18.5
4444.3	616	F	19.9	18.9	19.1	19.9	19.9+	19.8	18.8	18.5
4493.4	625	F	19.7	19.7	19.4	19.55	20.0	20.0	18.7	19.35
4709.6	669	F	19.0	19.15	19.1	19.8	19.7	19.8	18.75	18.2
4731.5	D323	F	19.7	19.7	19.7+	19.9	19.9	19.8	19.3	19.3
4739.5	S541	F	19.8	19.6+	18.9	19.5	19.7	19.0	19.15	18.25
4762.5	H676	G	19.3	19.0	19.2	19.7	19.8	19.7	19.15	18.5
4770.5‡	691	F	19.8	19.6	19.6	19.8	19.2	19.5	19.5	18.8
4771.5‡	697	F	19.9	19.7	19.6	19.45	19.4	19.6	19.5	18.9
4772.5	707	P	.....	.....	.....	.....	.....	19.5	.....	19.1
4800.4	709	G	19.6	19.4	19.55	19.8	19.8	19.8	19.5	19.2
4907.3	714	F	19.8	19.3	19.6	19.8+	20.0	19.9	19.15	19.2
4947.3	S556	P	.....	.....	.....	.....	.....	18.9	.....	.....
5035.63	H781	VP	.....	.....	.....	.....	.....	.....	.....	.....
5053.60	794	P	.....	.....	19.5	19.4	19.2	19.0	.....	19.3
5063.61	799	G	20.0	19.3	19.5	19.9	19.7	19.6	19.4	18.6
5065.68	810	G	19.8	19.4	19.6	20.0	19.2	19.8	19.5	18.7
5083.55	813	P	19.3	19.4	19.5+	19.6+	.....	19.45	19.4	18.9
5091.6	816	F	19.7	19.6	18.7	19.55	19.5	19.9	19.5	18.8
5115.5	825	F	20.0	19.3	19.5	20.0	19.5	19.6	19.4	18.4
5116.48‡	829	E	19.75	19.4	19.6	20.0	19.7	19.8	19.4	18.5
5124.45	H833	F	19.7	19.7	19.6	19.4	19.4	20.0	19.2	19.2
5145.52	S578	P	.....	.....	19.1	.....	.....	.....	19.0	18.9
5147.5‡	H843	F	20.0	18.8	19.1	19.7	19.6	19.8	19.1	18.9
5148.3	850	P	.....	18.8	19.2	.....	.....	.....	.....	18.9
5149.5‡	851	F	20.0	19.05	19.3	19.8	19.15	19.8	19.3	19.1
5150.5‡	858	F	19.8	19.15	19.45	19.9	19.3	20.0	19.4	19.2
5266.3	899	P	19.8	.....	18.7	19.5	.....	19.2	18.8	19.4
5475.5	920	G	19.9	19.6	19.45	19.9	20.1	19.4	19.4	19.2

TABLE II

Variables in M 31, Region 4

J.D.	29	30	31	32	33	34	35	36	37	38
4032.4	19.7	19.9+	19.2	19.4+	18.7	18.8	19.6	19.6	19.9	19.6
4054.4	19.2	19.5	19.2	19.6	18.9	19.8+	19.6	19.3	19.35	19.5
4056.4	19.8	18.9	19.5	19.5	18.95	19.8+	19.6	19.1	19.65	19.8
4058.55	19.6+	18.8	19.7	19.4	19.1	19.6+	19.4+	19.3	19.8+	19.9
4085.6	19.9+	19.8	19.8	19.5	19.0	19.9	19.6	19.7	20.0	19.7
4090.55	19.6+	19.4+	19.9	18.7	18.9	19.5+	19.6	19.1	19.8	19.7+
4108.35	19.7+	19.4	20.0	19.5	18.75	19.55	19.5+	19.1	19.5	19.8
4113.4	19.5	19.05	19.4+	19.3	19.1	19.6+	19.0	19.4	19.6+	.....
4115.5	19.5	19.2	20.0	18.7	19.3	19.6	19.3	19.5	19.9	19.6
4117.5	19.6+	19.6+	19.8+	19.0	19.4	.....	.....	19.6	19.8+	19.8+
4147.4	19.6+	18.75	19.4	19.2	18.9	19.6+	18.9	19.2	19.4	.....
4151.4	19.5	19.3	19.8	19.4	19.1	19.6+	19.4	19.6	19.9	19.9
4168.3	.....	18.8	.....	19.0	19.0	19.4+	19.3	.....	.....	.....
4171.3	19.4	19.3	19.65	19.1	19.2	19.5	19.5	19.6	19.8	20.0+
4180.3	19.8+	19.4	19.8+	19.4	19.0	19.7+	19.5+	19.1	19.9	19.9
4198.3	.....	.....	.....	.....	19.2	.....	.....	.....	.....	.....
4320.6	19.8+	19.8	19.9	18.9	19.1	19.6	.....	19.6	19.4	19.9
4323.6	.....	19.8	19.8	19.2	19.2	18.7	19.0	19.0	19.6	19.7+
4349.6	19.7	18.75	19.1	19.3	18.95	19.8	19.6+	19.5	19.8	19.8
4353.6	.....	19.1	19.5	19.4	18.75	19.3	19.4+	19.7	19.8+	19.7+
4357.5	19.9+	19.6	19.8	19.6	18.9	18.9	18.95	19.6	19.9	19.3
4377.5	19.9+	19.9	19.6	19.3	19.1	19.9	19.2	19.2	19.9	19.3
4379.5	.....	.....	.....	.....	19.2	.....	.....	.....	.....	.....
4384.5	19.4	18.7	20.0	.....	19.4	19.6	19.6	19.6	19.8	19.9
4387.45	19.8	18.8	20.0	19.6	19.0	18.7	19.7	19.7	19.4	19.4
4405.4	19.3	18.9	19.8	19.5	19.2	19.8+	19.6	19.6	19.8	19.8
4414.4	19.9+	19.8	19.9	19.4	19.0	19.8	19.4	19.2	19.4	20.1
4444.3	19.2	19.2	19.5	18.9	19.1	19.8	19.2	19.7	19.9	19.9+
4493.4	19.9	19.4	19.9	19.4	19.3	19.4	19.6	19.6	19.35	19.8
4709.6	19.9+	19.9	20.0	19.4	18.6	18.9	19.15	19.6	19.55	19.8
4731.5	19.9+	19.1	20.0	19.1	18.9	19.9	19.5	19.6	19.7	19.5
4739.5	19.6+	19.2	19.3	19.4	19.3	19.6+	19.6	19.4	19.9	19.8
4762.5	20.0+	19.9+	19.3	19.5	19.3	19.9	19.2	19.6	19.55	19.35
4770.5‡	20.0+	18.8	20.0	19.6	19.1	19.8	19.6	19.5	19.7	19.8
4771.5‡	19.8+	19.0	19.9	19.5	19.0	19.55	19.6	19.0	19.7	19.6
4772.5	.....	19.15	19.5+	19.4	19.1	.....	.....	19.0	19.3	19.5
4800.4	19.9+	19.8	19.9	18.8	19.3	19.8	19.4	19.7	19.5	19.9
4907.3	19.9+	19.7+	19.9	19.2	19.3	19.2	19.5	19.6	19.4	19.8
4947.3	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
5035.63	.....	.....	.....	.....	19.1	.....	.....	.....	.....	.....
5053.60	.....	19.3+	19.2	18.75	19.1	.....	.....	.....	.....	.....
5063.61	20.0+	18.95	19.9	19.4	19.3	19.05	19.4	19.65	19.9	19.7
5065.58	20.0+	19.1	19.9	19.4	19.2	19.05	19.6	19.05	19.6	19.4
5083.55	19.4+	19.1	19.6	19.0	19.3	19.4+	.....	19.6+	19.7	19.8
5091.6	19.5	19.6	19.9	19.5	18.95	19.7	19.6	19.6	19.9	19.9+
5115.5	19.8+	19.2	20.0	19.5	19.4	19.7	19.5	19.5	20.0	19.6
5116.48‡	19.9+	18.65	19.9	19.6	19.4	19.8	19.6	19.6	20.0	19.4
5124.45	19.6	19.7	19.3	19.5	18.6	19.8	19.6	19.8	19.9	19.7
5145.52	19.5	19.6+	.....	.....	18.9	.....	18.9	19.5	19.6	.....
5147.5‡	19.4	19.7	19.3	19.5	18.95	19.7	19.3	19.1	19.4	19.3
5148.3	.....	.....	.....	.....	18.9	.....	.....	19.2	.....	.....
5149.5‡	19.6	19.65	19.5	19.5	19.15	19.8	19.4	19.4	19.6	19.75
5150.5‡	19.7	19.55	19.6	19.5	19.2	19.8	19.5	19.5	19.7	19.9
5266.3	.....	18.8	.....	19.2	19.4	.....	.....	19.5+	19.4	.....
5475.5	19.8	19.7	19.8	19.7	19.4	19.8	19.6	19.5	19.8	19.8

TABLE II—*Continued*

J.D.*	Plate†	Qy.	39	40	41	42§	43	44	45	46
4032.4	H424	F	19.7	19.0	19.6	19.0	19.8	19.2	19.6	19.7
4054.4	432	P	19.8+	19.0	19.3	18.5	19.7	19.1	20.0	19.6
4056.4	S379	G	19.9+	19.1	19.6	18.4	19.8+	18.7	19.7	19.7
4068.55	H438	F	.....	18.9	19.6	18.2	19.7	19.1	19.3	19.8
4085.6	454	E	19.3	19.0	19.7	17.9	19.7	19.3	19.35	19.7
4090.55	461	P	.....	18.8+	19.6+	17.9	19.7	19.2	19.7+	19.4
4108.35	468	F	19.1	18.6	19.4	18.4	19.7+	19.1	19.9	19.8
4113.4	S408	P	.....	.....	19.4+	18.4	.....	19.0	19.7	19.6
4115.5	H478	F	19.7	19.0	19.7	18.5	19.8+	19.2	19.9	19.4
4117.5	497	F	19.5+	19.0	19.5+	18.4	19.7	19.2	19.7+	19.5
4147.4	504	P	19.5+	18.9	19.4	19.0	19.6	18.7	19.7	19.7
4151.4	512	F	19.8	19.15	18.9	19.1	19.7	18.7	19.7	19.7
4168.3	S416	P	.....	.....	.....	19.0	.....	.....	.....	.....
4171.3	H516	G	19.8	18.4	19.6	19.0	19.9	18.7	19.9	19.7
4180.3	525	F	19.2	18.75	19.0	19.0	19.7+	19.0	19.9	19.2
4198.3	S439	P	.....	.....	.....	19.1	.....	.....	.....	.....
4320.6	H573	F	19.6	19.1	19.6	18.8	19.5	18.7	19.7	19.45
4323.6	S494	F	19.2	19.1	19.5	18.9	.....	18.7	19.9+	19.35
4349.6	H581	G	19.45	18.9	19.7	19.0	19.4	18.7	20.0	19.4
4353.6	D257	P	.....	18.8+	18.9	19.0	19.3	18.8	19.9	19.7
4357.5	H583	G	19.9	19.2	18.95	18.9	19.3	18.8	19.7	19.6
4377.5	587	E	19.7	18.7	19.2	18.9	19.35	18.8	19.95	19.6
4379.5	599	P	.....	18.75	18.7	19.0	.....	.....	.....	.....
4384.5	D276	E	20.1	18.9	19.1	18.9	19.4	18.8	19.8	19.6
4387.45	287	E	20.0	19.2	19.4	18.85	19.4	18.9	20.0	19.3
4405.4	H605	G	20.0	18.4	18.8	18.3	19.5	18.7	19.8	19.8
4414.4	610	F	20.0	18.8	19.4	17.9	19.45	18.8	20.0	19.3
4444.3	616	F	19.3	18.75	19.7	18.1	19.6	18.7	19.8	19.7
4493.4	625	F	19.45	19.1	19.5	19.0	19.7	18.8	19.9	19.5
4709.6	669	F	19.5	18.6	19.2	19.1	19.3	19.3	20.0	19.3
4731.5	D323	F	19.15	19.2	19.7	19.0	19.1	19.4	19.5	19.8
4739.5	S541	F	19.8	18.5	19.0	19.2	19.2	19.3	19.8+	19.5
4762.5	H676	G	19.9	19.15	18.95	18.3	18.9	19.3	20.0	19.2
4770.5‡	691	F	19.8+	18.8	19.6	18.1	19.1	19.1	19.45	19.7
4771.5‡	697	F	20.0	18.5	19.6	18.0	18.9	19.1	19.6	19.6
4772.5	707	P	.....	18.3	19.6	18.0	.....	19.1	.....	.....
4800.4	709	G	19.5	19.2	19.7	18.3	19.0	19.1	19.9	19.25
4907.3	714	F	19.7	18.4	19.6	19.0	19.7	19.2	20.0	19.5
4947.3	S556	P	.....	.....	.....	18.2	.....	.....	.....	.....
5035.63	H781	VP	.....	.....	.....	19.0	.....	.....	.....	.....
5053.60	794	P	.....	18.9	.....	19.1	.....	.....	.....	.....
5063.61	799	G	19.6	19.1	19.6	19.2	19.75	19.25	19.7	19.6
5065.58	810	G	19.0	19.1	19.5	19.2	19.75	19.2	19.3	19.8
5083.55	813	P	19.5+	18.75	19.5+	19.1	.....	.....	19.9	19.5
5091.6	816	F	19.35	19.1	19.3	18.8	19.5	19.3	19.5	19.8
5115.5	825	F	19.4	18.7	19.7	18.2	19.4	19.2	19.7	19.7
5116.48‡	829	E	19.5	18.75	19.5	18.2	19.3	19.1	19.2	19.9
5124.45	H833	F	19.8	19.2	19.15	17.8	19.25	19.2	20.0	19.35
5145.52	S578	P	.....	18.6	18.8	17.9	19.2	19.2	.....	.....
5147.5‡	H843	F	19.8+	18.7	19.0	18.1	19.0	19.2	19.9	19.55
5148.3	850	P	.....	18.8	19.1	18.1	.....	.....	.....	.....
5149.5‡	851	F	19.9	18.8	19.25	18.2	18.9	19.1	19.9	19.25
5150.5‡	858	F	19.9	18.85	19.4	18.25	18.9	19.1	20.0	19.25
5266.3	899	P	.....	19.1	19.5	18.8	19.5	19.2	19.5+	19.2
5475.5	920	G	19.7	18.4	18.8	18.0	19.8	19.4	19.9	19.5

* Add 2,420,000.

† D plates were obtained by Duncan; S and H plates, by Hubble.

‡ Two or more plates in one night.

|| The plus sign following a magnitude indicates that the variable was invisible, but certainly fainter than the tabulated magnitude.

§ Additional observation of Variable No. 42, H916, J.D. 5291.3, 17.9.

Cepheids.—Data for the normal light-curves of the forty variables recognized as Cepheids are given in Table III. Magnitudes at minima and ranges are included, but in most cases they are very uncer-

TABLE III

CEPHEIDS IN M 31

VAR. NO.	PERIOD IN DAYS	LOG P	PHOTOGRAPHIC MAGNITUDES				EPOCH J.D.*	POSITION†	
			Max.	Min.	Range	Med.		X	Y
1.....	31.390	1.497	18.2	19.4	1.2	18.8	3823.0	+13.7	+ 8.1
2.....	20.090	.303	18.6	(19.6)‡	(1.0)	(19.1)	4029.5	− 1.6	−13.9
3.....	27.00	.431	18.4	19.7	1.3	19.05	4771.0	+ 8.0	− 4.3
4.....	20.60	.314	19.0	(19.8)	(0.8)	(19.4)	3995.5	− 1.4	+ 5.0
5.....	48.36	.684	18.5	(19.7)	(1.2)	(19.1)	3990.0	+21.6	−10.6
6.....	22.23	.347	18.6	19.7	1.1	19.15	3788.0	−10.7	− 9.7
7.....	45.05	.654	18.1	19.1	1.0	18.6	2583.0	−16.9	− 3.2
8.....	17.764	.249	19.0	(19.8)	(0.8)	(19.4)	3992.5	− 4.2	+ 2.0
9.....	32.56	.513	18.2	19.4	1.2	18.8	4108.0	−23.1	+ 1.4
10.....	22.823	.358	18.8	(19.6)	(0.8)	(19.2)	4029.0	+24.2	−15.3
12.....	22.225	.347	18.9	(20.0)	(1.1)	(19.45)	3994.5	−18.3	− 3.2
13.....	21.726	.337	19.05	(19.95)	(0.9)	(19.5)	3822.0	−16.7	− 6.8
14.....	17.632	.246	18.9	(20.0)	(1.1)	(19.45)	4026.0	− 9.8	− 7.3
16.....	41.157	.614	18.8	19.8	1.0	19.3	4011.0	+11.9	−10.5
17.....	18.773	.274	18.75	19.9	1.15	19.3	3990.0	+14.3	− 8.0
18.....	18.538	.268	19.1	(19.9)	(0.8)	(19.5)	4025.0	+11.5	− 7.3
21.....	17.15	.234	19.0	20.0	1.0	19.5	4055.0	+51.0	− 0.9
22.....	17.60	.246	18.95	(19.85)	(0.9)	(19.4)	4181.0	46.7	+ 0.5
23.....	17.54	.244	18.7	19.7	1.0	19.2	4319.0	54.5	− 4.1
24.....	17.61	.246	19.25	(20.0)	(0.75)	(19.6)	4349.0	49.7	− 3.2
25.....	11.82	.073	19.1	19.9	0.8	19.5	4511.0	49.2	3.6
26.....	26.18	.418	19.0	19.9	0.9	19.45	4319.0	46.6	3.6
27.....	24.07	.381	18.75	19.45	0.7	19.1	4324.0	49.4	5.8
28.....	26.78	.428	18.25	19.35	1.1	18.8	4147.0	51.7	6.9
29.....	19.524	.290	19.1	(20.0)	(0.9)	(19.55)	4444.0	49.4	6.7
30.....	18.28	.262	18.75	19.85	1.1	19.3	4383.0	44.5	4.9
31.....	22.73	.357	19.1	20.0	0.9	19.55	4349.0	48.0	2.1
32.....	25.32	.403	18.7	19.6	0.9	19.15	4318.0	61.9	6.0
33.....	18.82	.275	18.7	19.3	0.6	19.0	4031.0	54.0	1.9
34.....	32.25	.508	18.8	19.9	1.1	19.35	4708.0	42.0	9.4
35.....	17.49	.243	18.95	(19.6)	(0.65)	(19.3)	4356.0	34.3	4.2
36.....	17.90	.253	19.05	(19.75)	(0.7)	(19.4)	4325.0	53.0	8.4
37.....	13.328	.125	19.3	(19.9)	(0.6)	(19.6)	4319.5	52.6	2.6
38.....	10.128	.005	19.3	(20.0)	(0.7)	(19.65)	4357.0	51.2	5.1
39.....	23.92	.379	19.1	(20.1)	(1.0)	(19.6)	4730.0	46.5	7.4
40.....	33.455	.524	18.4	19.2	0.8	18.8	4373.0	48.0	3.5
41.....	25.458	1.406	18.8	19.7	0.9	19.25	4379.0	56.7	5.1
42.....	175.	2.243	17.9	19.2	1.3	18.55	4425.0	58.1	2.4
45.....	12.90	1.111	19.3	(20.0)	(0.7)	(19.65)	4084.0	51.9	8.0
46.....	12.93	1.112	19.2	(19.8)	(0.6)	(19.6)	4321.5	+52.4	−1.4
Mean..	.....	1.350	18.81	.....	.....	.....	.....	.....	.....
Mean, omitting No. 42	.....	1.327	18.83	.....	.....	.....	.....	.....	.....

* Add 2,420,000 to tabular values.

† Positions are referred to the nucleus, X along the major axis, + to the south preceding; Y along the minor axis, + to the north preceding.

‡ Magnitudes in parentheses are uncertain.

tain and are bracketed in order to indicate this fact. Positions are given to the nearest 0.′1 with respect to a system of co-ordinates determined by the major and minor axes of the nebula, and the stars themselves are marked on the various plates used as illustrations. The same system is used for the novae and will be described more fully in connection with the distribution of those objects. For the present it is sufficient to mention that Cepheids are rare in the un-

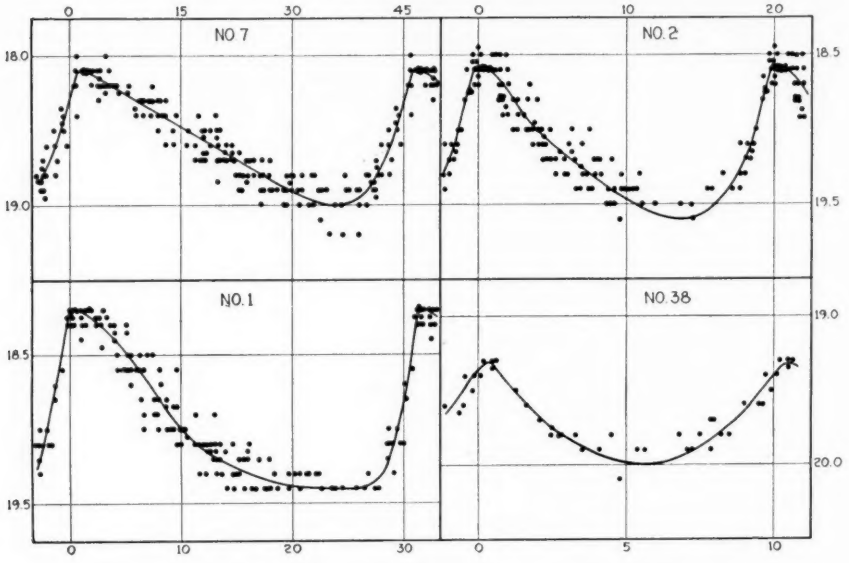


FIG. 1.—Light-curves of four Cepheids in M 31; ordinates, photographic magnitudes; abscissae, days.

resolved nuclear region and that position in the nebula appears to have no effect on the period-luminosity relation.

The periods of the Cepheids range from forty-eight to ten days, and the magnitudes at maxima, from 18.1 to 19.3, with the exception of No. 42, which varies from 17.9 to 19.2 in a period of one hundred and seventy-five days. The Cepheid characteristics are obvious in the normal curves illustrated in Figure 1, and the forms of the curves tend to harmonize with Hertzsprung's relation between shape and period.[1]

The numbers of Cepheids increase steadily as the periods shorten,

[1] *Bulletin of the Astronomical Institute of the Netherlands*, **3**, 115 (No. 96), 1926.

with a limit at about seventeen days. This appears to represent the general limit of the data, and the five variables with shorter periods may be regarded as chance catches. Other very faint variables have been found, but the very exacting conditions required for determining periods have not as yet been realized. There are indications, however, that the periods of some at least are less than ten days.

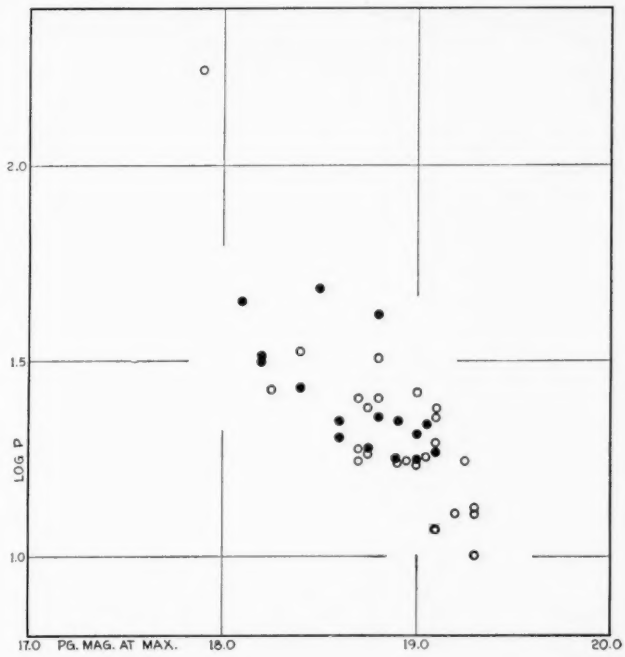


FIG. 2.—Period-luminosity relation among Cepheids in M 31. Photographic magnitude at maximum plotted against logarithm of period expressed in days. Cepheids in Region 4 are designated by circles in order to emphasize the absence of any selective effect due to position in the nebula.

In Figure 2, the logarithms of the periods have been plotted against magnitudes at maxima. The variables in Region 4 have been distinguished by circles in order to emphasize the absence of any effect that can be attributed to location in the nebula. The relatively larger number of very faint variables in the outer region is probably due to the longer exposures used in studying that field.

The period-luminosity relation is clearly shown, although the

range is limited and the data for the fainter magnitudes are incomplete. The slope is approximately that of the curve published by Shapley for Cepheids in the Small Magellanic Cloud.[1] The residuals, which average about 0.2 mag., are slightly smaller than those found by Shapley, but the difference can be attributed largely to the incompleteness of the data for the fainter variables. Number 42, the outstanding exception, deviates from the curve by 0.75 mag., but the next largest residual, that for No. 16, is less than 0.5 mag.

The distance of M 31 has been derived by comparing Figure 2 with corresponding diagrams for Cepheids in M 33 and in the Small Magellanic Cloud. The latter diagrams, drawn on transparent paper, were superposed on Figure 2 and shifted along the axis of magnitudes until the best fit was obtained. Some personal judgment was involved in allowing for the effects of selection in favor of brighter stars among the shorter periods, but the limits of the probable errors are small.

In the case of the two spirals, where the scales of magnitudes are strictly comparable, since they are based largely upon the same Selected Areas, the shift in m, the apparent magnitude, was determined as 0.1 mag., in the sense that the stars in M 31 are the fainter. A shift of 0.1 mag. in either direction from this adopted value appreciably unbalanced the fit, hence the probable error of the determination was estimated to be of the order of 0.05 mag.

M 33 had already been compared with the Small Magellanic Cloud in order to determine the relative distance, hence the comparison of M 31 with the cloud was largely a check on the previous comparison. The result for the new comparison, $\Delta m = 4.7 \pm 0.1$, agrees, within the probable errors, with the value for M 33 when corrected for the difference between the two spirals.

The data for the two spirals are so nearly comparable—thirty-five Cepheids being known in M 33 and forty in M 31—that they may be combined into a single diagram for a final comparison with the diagram for the Small Magellanic Cloud. The result is shown in Figure 3. The magnitudes for the Cepheids in M 33 have been increased by 0.1 mag. in order to reduce them to the distance of M 31. These seventy-five variables form a period-luminosity diagram of

[1] *Harvard Circular*, No. 280, 1925.

considerable weight, in which the internal relations are reasonably consistent.

To obtain the best fit, the superposed diagram for the Small Cloud was shifted 4.65 mag., with an estimated probable error of 0.1 mag. The magnitudes at maximum of the Cepheids in the Small Cloud, increased by 4.65 mag., are included in Figure 3 in order to show the appearance of the diagram for what was judged to be the best agreement. For the sake of completeness, nine Cepheids in N.G.C. 6822 have also been included, their magnitudes increased by 0.55 mag.[1] Figure 3, therefore, includes all extra-galactic Cepheids for which data have been published.

No corrections to the distances of N.G.C. 6822 and M 33 as previously determined are indicated. The distance of M 31 is about 0.1 mag., or 5 per cent, greater than that of M 33, and 8.5 times the distance of the Small Magellanic Cloud. Using Shapley's value for the cloud ($m-M=17.55$), we find for M 31

$$m-M=22.2$$
$$\pi=0''.00000363$$
$$\text{Distance}=275{,}000 \text{ parsecs}$$
$$=900{,}000 \text{ light-years}$$

[1] The periods of the Cepheids in N.G.C. 6822, as published in *Mt. Wilson Contr.*, No. 304; *Astrophysical Journal*, **62**, 409, 1925, were derived from rather limited material. Additional data collected in the last two years have led to some revision. The revised periods, which are believed to be reliable, are as follows:

No. 7	65.05 days	No. 4	17.36 days
2	37.50	9	16.84
1	30.47	5	13.864
3	29.24	13	122.
6	20.00		

The possibility of periods close to an even day or submultiple thereof has definitely been eliminated in the case of variables Nos. 2, 4, 5, and 6 by series of morning and evening plates on successive nights. Numbers 10 and 12 appear bright and faint in alternate seasons and probably have periods of the order of six hundred days. Their color-indices are estimated as moderately large, but the stars are not red. No revisions have been found necessary for Nos. 8 and 11, but the data in these cases are very uncertain. Two new variables have been found, both of which appear to be irregular. One of these stars is very red.

Continued observation of M 33 has led to the discovery of two additional novae, making four in all, and of two new variables, but to no revision in the periods of the Cepheids. Dr. W. Baade, of the Hamburg Observatory, has communicated by letter his discovery of an additional variable in the extreme northern region of M 33. This star appears near the edge of several of the Mount Wilson plates, where the variation, although small, is quite definite. It is one of the brightest variables in the region, being surpassed only by Nos. 1 and 2.

The accuracy of the relative distances is very satisfactory. In the case of the two spirals, the probable error is of the order of 2.5 per cent; for the spirals and the Cloud, it is of the order of 5 per cent. The accuracy of the distances in parsecs or light-years, however, depends largely upon the accuracy of the zero point of the period-

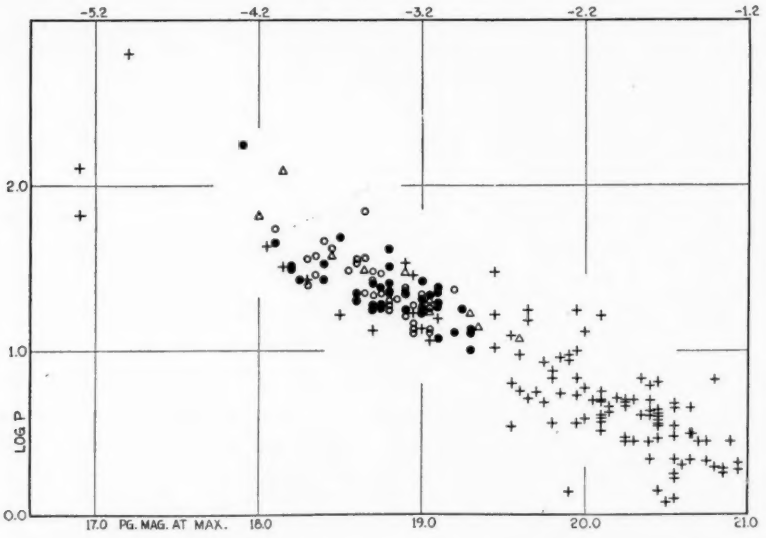


FIG. 3.—Period-luminosity relation among the extra-galactic Cepheids. The crosses refer to 106 Cepheids observed by Shapley in the Small Magellanic Cloud; the black discs, to 40 Cepheids in M 31; the open circles, to 35 in M 33; the triangles, to 9 in N.G.C. 6822. The apparent magnitudes at maxima have been reduced to the distance of M 31 by adding 4.65 to those in the Small Magellanic Cloud, 0.1 to those in M 33, and 0.55 to those in N.G.C. 6822. The absolute photographic magnitudes at the top of the diagram are based upon Shapley's zero point ($m-M=17.55$ for the Small Magellanic Cloud).

luminosity curve. Accumulating evidence indicates that Shapley's value is certainly of the right general order of magnitude, but there still remains the possibility of a considerable correction when more data on galactic Cepheids become available.

Variables other than Cepheids.—Of the ten variables other than recognized Cepheids, four are probably Cepheids for which the data are insufficient to establish the characteristics, while six are irregular or have long periods. To the first class belong the following stars:

Variable No.	Max.	Min.	X	Y
47.....	19.7	20.3	+55.5	−3.9
48.....	19.6	20.3	48.8	4.0
49.....	19.6	20.3	49.7	4.4
50.....	19.2	(19.5)	+50.0	−6.2

Number 50 is involved in a group or cluster, and is especially difficult to observe. These stars can be followed, in general, only on plates exposed under exceptionally good conditions. Short periods, however, may be inferred from the short duration of maxima. In the case of No. 47, two maxima have been observed with an interval of five days.

The irregular or long-period variables are Nos. 11, 15, 19, 20, 43, and 44.

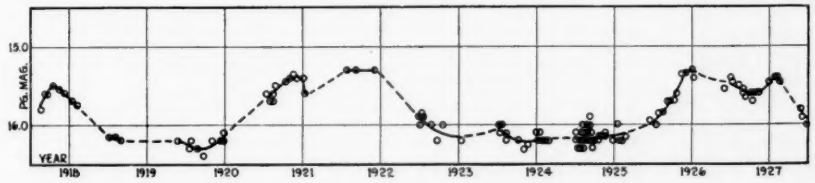


FIG. 4.—Light-curve of variable No. 19

Number 11 ($X = -7'.5$; $Y = -8'.3$) has varied from 18.6 to about 19.6. The brightest maximum occurred in the autumn of 1917, and seven years later it again rose above 19.0.

Number 15 ($X = -16'.1$; $Y = -10'.4$), the second brightest variable in the nebula, rose gradually from 17.5 in 1909 to 16.8 in 1924. In 1926 it faded to 17.3, but rose again to 17.0 in 1927 and in 1928, to about 16.7. The absolute magnitude at maximum, calculated with the distance indicated by the Cepheids, is about −5.5. The color-index appears to be less than 0.2 mag.

Number 19 ($X = -2'.3$; $Y = -9'.8$) is the brightest of all the variables that have been observed. Its color-index is uncertainly estimated as of the same order as that of No. 15—less than 0.2 mag. The variation is from about 15.3 to 16.3, and there are some indications of a period of the order of five years. The light-curve, however, which is shown in Figure 4, does not repeat itself very faithfully, and further observations will be required to settle the question. If the

star actually belongs to the nebula, the absolute magnitude varies from −6.9 to −5.9. Numbers 15 and 19 show some analogies with variables Nos. 1 and 2 in M 33. Number 19 was found independently by W. J. Luyten on plates made with the 24-inch Bruce camera.[1]

Number 20 ($X = +4'.6$; $Y = +11'.5$) was recorded on several plates from 1909 to 1910 as about 18.6. By 1915, however, it had risen to about 18.0, where it has remained up to the present. For this reason the individual observations are not listed in Table I. The star may be similar to those listed as novae Nos. 36 and 39, which appeared suddenly but have remained constant since their appearance. It may be significant that these are among the faintest novae that have been observed.

Numbers 43 ($X = +51'.3$; $Y = -4'.8$) and 44 ($X = +43'.5$; $Y = -8'.1$) are in Region 4 and hence have been observed only since 1924. The former appears to be red, while the latter has a color-index considerably less than the color-indices of the Cepheids. Number 43 averaged about 19.8 in 1924–1925 and about 19.4 in 1925–1926. From 19.2 in August, 1926, it rose to 18.9 in September, only to fade again until by January, 1927, it was 19.6. In July it was still faint, about 19.7, but rose rapidly to 18.9 in October. By January, 1928, however, it had faded to 19.5, and by August had reached 19.8.

Number 44 has varied from 18.7 to about 19.4. It was active during the first season, 1924–1925, rising, fading, and rising again to the maximum observed luminosity. During 1925–1926 it averaged about 18.8, and since then, about 19.2.

The significant feature of these stars appears to be their restricted ranges in luminosities. They are giant stars and might well be classed as "super-giants." One, No. 43, is clearly red, but three at least appear to be early-type stars. The red star may be analogous to Betelgeuse, but stars similar to the other three variables are not known in our own system. It is not improbable, however, that the variables found in M 81 and N.G.C. 2403, and some at least of those in M 101, are similar to the bright irregular variables in M 31 and 33, and may eventually be used as rough criteria of distances.

[1] *Harvard College Observatory Bulletin*, No. 851, 1927, in which Luyten announces another variable, about 13.5 at maximum, on the extreme edge of M 31, north following the nucleus. Color and type of variation are uncertain, but in view of the relative luminosity, further data will be required to determine a connection with the spiral.

NOVAE

S Andromedae, the first nova found in the spiral, was discovered visually in August, 1885. It was announced by Hartwig, although, as subsequently disclosed, it had been seen previously by several other observers. The position of this nova, about 16″ preceding and 4″ south of the nucleus, is unique. No other has been found within 1′.5 of the nucleus, although it should be remembered that only exceptionally bright novae could be detected on the dense background of unresolved nebulosity in this region. The photometric measures of the first nova are rather discordant, but a general review of the published results indicates that the visual magnitude at maximum was about 8.0. This places it at once in that mysterious class of exceptional novae which attain luminosities that are respectable fractions of the total luminosities of the systems in which they appear.

Photographic observations of novae in M 31 began in 1917. Ritchey had found a nova in N.G.C. 6946 and, following up the suggestion, examined all the negatives of spirals in his collection. Indications of changes, novae or variables, were reported in M 81, M 101, and N.G.C. 2403, but no further investigations were made of these objects.[1] In M 31, however, two novae were found on the first plates of the nebula obtained with the 60-inch reflector in the autumn of 1909, and sufficient data were available to establish the character

[1] *Publications of the Astronomical Society of the Pacific*, **29**, 210, 1917. Ritchey's remarks refer to the following objects: (*a*) The star in M 81 is 4′.5 from the nucleus in a direction S 27° W. On two plates taken in February, 1910, it was about 19.0; on three plates in March, 1916, and March and December, 1917, it was about 18.6; in April, 1921, and from 1924 to 1927 it was normal again at 19.0. Six additional variables have since been found, and all appear to be analogous to the brighter irregular variables in M 31. (*b*) The object in N.G.C. 2403 appears to be a defect. Under a microscope the edges are sharper and the color more brownish than the ordinary star-images. Five variables are now known in this nebula, but the nature of the variations is uncertain. (*c*) In M 101, two stars were marked. One, 2′.85 due south of the nucleus, was about 18.8 in March, 1910, and about 19.5 in May, 1915. On a few subsequent long exposures it appears about the latter luminosity. The other star, 4′.13 from the nucleus in the direction S 4° E, was about 19.2 in 1910 and 19.6 in 1915. From 1922 onward it has remained constant at about 18.5. Eight other variables have since been found.

The 1910 plate of M 101 shows a star of 16.8 mag., some 12′.3 from the nucleus in the direction W 35° N, which does not appear on any of the later plates. It was not mentioned by Ritchey, although the image shares the distortion, due to coma, of the other images and there seems to be no reason for doubting that it represents an actual star. It might well be an unusually bright nova.

of the variation. Other plates of the spiral were immediately obtained, and these led to the discovery of six additional novae during the season 1917–1918, by Ritchey, Shapley, and Duncan. During the next five years the spiral was photographed more or less sporadically, and thirteen novae were discovered by Sanford, Humason, Duncan, Shapley, and Miss Ritchie.

In the autumn of 1923 the writer began to photograph the nebula more systematically in order to accumulate data for the statistical study of novae. Since that time sixty-three additional novae have been found, and the material is now sufficient for a preliminary general discussion.[1] The nova of 1885 is clearly an exceptional case, and the eighty-five photographic novae must be considered as normal.

Although more than three hundred plates are now available, the material is not strictly homogeneous with respect either to effective exposure or to spacing in time. The 60- and 100-inch reflectors have been used indiscriminately under all conditions of seeing and with exposures ranging from twenty minutes to upward of two hours. The limiting magnitudes on the various plates are usually 19.0 or fainter, although occasionally they may be brighter than this by 0.5 mag. or more. A hundred plates, however, show stars fainter than 20.0. The observing seasons ran from June to February. The exposures were all made during the dark of the moon. From first quarter to third quarter both reflectors are always used in the Cassegrain form with the spectrographs attached, and hence are not available for direct photography. As a result, at least half of each lunation was unobserved, and generally the remaining half was not fully covered. For the years before 1923, the gaps are considerably greater. Following Ritchey's excellent series in the autumn of 1909, only four plates were exposed during an interval of eight years.

In general, the plates were centered on the nucleus and, as regards limiting magnitudes, the distortion of the images in the outer regions is roughly balanced by the background of unresolved nebulosity near the nucleus. Since the plane of the spiral is but little in-

[1] The four novae, Nos. 87–90, discovered by Duncan in August, 1928, are not included in the present discussion. The announcement is in *Publications of the Astronomical Society of the Pacific*, **40**, 347, 1928.

clined to the line of sight, a strip nearly 30′ wide, extending entirely across the nebula at right angles to the major axis, has been observed in a fairly homogeneous manner. In addition, Region 4, with its center some 48′ south preceding the nucleus along the major axis, has been under observation since 1924; Regions 2 and 3 were well observed during the season 1924–1925; about thirty plates were centered on other regions or covered large areas with long exposures. These furnish information concerning the outer regions of the spiral supplementary to that contained in the main series centered on the nucleus.

Following the procedure established in 1917, the novae have been numbered serially according to the date of discovery. Number 1 is therefore the great nova of 1885, S Andromedae; Nos. 2–22, inclusive, are the photographic novae found at Mount Wilson from 1917 to 1922; Nos. 23 and upward are the novae found by the writer from 1923 to 1927. Data on Nos. 2–21 have been published from time to time in *Publications of the Astronomical Society of the Pacific*.[1] The magnitudes were generally estimated by comparison with a sequence of stars measured by Shapley with the 60-inch telescope. These measures have not been published, but are on file at Mount Wilson.

New measures by the writer with the 100-inch are in fair agreement down to about 17.0, but beyond this point they diverge somewhat, probably because of the greater efficiency of the larger telescope in registering star-images free from the nebulous background. For the sake of homogeneity, the plates of the older novae have been re-examined and the magnitudes have been re-estimated by comparison with the new sequence. These data, together with those for the more recent novae, are given in full in Table IV. The date of the plate immediately preceding that on which a nova first appeared and the date of the plate next following that on which it was last

[1] The references are as follows:

Novae	2, 3	**29**, 210, 1917	
	4	**29**, 213, 1917	
	5	**29**, 257, 1917	
	6, 7, 8	**30**, 162, 1918	
	9	**30**, 255, 1918	
	10, 11	**30**, 341, 1918	
Novae	12, 13	**31**, 109, 1919	
	14, 15, 16	**31**, 280, 1919	
	17	**32**, 63, 1920	
	18, 19, 20	**33**, 56, 1921	
	21	**34**, 222, 1922	

TABLE IV

MAGNITUDES OF NOVAE IN M 31

1909	J.D.	Plate	Qy.	2‡	3	38				
Sept. 12....	8562.5*	R17	G	18.3+	18.2+					
12....	8562.7	18	F	17.6	17.5	18.7				
13....	8563.45	19	G	16.9	17.7	18.7				
13....	8563.65	20	G	17.4	17.7	18.7				
15....	8565.45	26	E	16.7	17.4	18.9				
16....	8566.65	27	F	16.8	17.1	18.8				
Oct. 11....	8591.45	34	E	17.4	17.6	18.8				
13....	8593.35	39	G	17.5	17.7	19.4				
Nov. 7....	8618.3	47	P	17.9	17.5	19.4+				
7....	8618.5	48	P	18.0	17.6	18.3+				
1917–18	**J.D.**	**Plate**	**Qy.**	**4**	**5**	**6**	**7**	**8**	**9**	**85**
Aug. 12....	1462.65†	P300	G	18.6+						
Sept. 11....	1483.45	Sh3991	P	17.5						
17....	1489.5	R146	G	17.7	18.5+					
Oct. 13....	1515.5	Sh4091	VP	18.2+	17.3					
16....	1518.5	R147	F	18.6	17.4	18.3+				19.0+
Nov. 14....	1547.5	151	G	18.5+	18.2	16.5	.....	.....	.....	17.4
Dec. 12....	1575.4	154	G	.....	18.5+	17.5	.....	.....	.....	17.4
13....	1576.4	156	G	.....	.....	17.6	18.6+	.....	.....	19.0+
Jan. 15....	1609.3	161	G	.....	.....	18.3	17.1	18.5+	18.5+	
17....	1611.3	Sh4333	VP	.....	.....	17.6+	17.1	17.6+	17.6+	
Feb. 9....	1634.3	R162	F	.....	.....	18.5+	18.5+	17.5	17.5	
10....	1635.3	163	F	.....	.....	.....	.....	17.4	17.5	
1918–19	**J.D.**	**Plate**	**Qy.**	**10**	**11**	**12**	**13**			
Aug. 30....	1836.6	Sh4663	P	18.0+	18.0+					
Oct. 7....	1874.4	Sa10	F	17.4	17.6					
8....	1875.36	Sa12	F	17.5	17.6					
8....	1875.38	13	F	17.5	17.7					
Nov. 2....	1900.4	17	G	17.8	18.8+					
2....	1900.4	18	F	17.8	.....	18.5+	18.5+			
Jan. 4....	1963.23	21	P	18.0+	.....	17.0	17.2			
4....	1963.24	22	P	.....	.....	17.1	17.2			
5....	1964.3	23	P	.....	.....	17.0	17.3			
Feb. 3....	1993.25	27	G	.....	.....	17.5	18.5			
3....	1993.3	28	G	.....	.....	17.5	18.5			
1919	**J.D.**	**Plate**	**Qy.**	**14**	**15**	**16**	**17**	**86**		
June 28....	2138.7	Sa36	P	.....	17.7					
29....	2139.7	37	G	.....	17.8					
30....	2140.6	38	G	.....	17.9					
30....	2140.7	39	F	18.8+	18.0					
July 21....	2161.6	Sh4978	F	16.1	17.4					
23....	2163.6	5025	F	16.1	17.5					
28....	2168.6	D24	P	16.2	17.4					
29....	2169.6	27	G	16.3	17.6	17.8+				
Aug. 24....	2195.5	30	F	18.8+	17.9	17.1				
28....	2199.5	Sa41	G	.....	18.2	17.1				
29....	2200.5	42	G	.....	18.0	17.1				
Sept. 1....	2203.5	46	P	.....	18.0	17.2				
21....	2223.5	Sh5054	P	.....	17.9	17.4	.....	16.7		
Oct. 16....	2248.4	Sa49	P	.....	18.0	17.8+	18.5+			
30....	2262.4	Sh5105	VP	.....	.....	.....	15.8			
31....	2263.4	5107	G	.....	18.0	.....	16.6			
Nov. 30....	2293.4	Sa50	F	.....	18.2	.....	18.5+			
Dec. 21....	2314.3	Sh5160	G	.....	18.2					
1920	**J.D.**	**Plate**	**Qy.**	**18**	**19**	**20**	**80**	**81**		
July 22....	2528.6	Sh5479	P	.....	.....	.....	17.8+			
Aug. 8....	2545.6	5489	G	.....	.....	.....	18.0			
10....	2547.63	D42	F	.....	.....	.....	17.9			
11....	2548.56	44	E	.....	.....	.....	18.0			
Sept. 12....	2580.5	62	F	.....	.....	.....	17.5			
13....	2581.6	65	P	.....	.....	.....	17.5			

TABLE IV—*Continued*

1920–21	J.D.	Plate	Qy.	18	19	20	80	81		
Sept. 16,17.	2585.0	71	G	18.5+	.....	.....	17.8			
17....	2585.3	72	P	.....	.....	.....	17.8			
Oct. 10....	2608.5	76	F	17.2	.....	.....	18.1			
11....	2609.6	80	P	17.3	.....	.....	18.2			
14....	2612.3	S68	F	17.4	.....	.....	18.1			
Nov. 12....	2641.6	75	F	18.3+	.....	.....	18.5			
13....	2642.5	77	G	18.5+	18.5+	18.5+	18.5	19.0+		
Dec. 10....	2669.3	D106	F	.....	16.0	17.7	18.6	16.7		
10....	2669.35	107	F	.....	15.9	17.7	18.6	16.7		
12....	2671.3	112	P	.....	16.6	17.6	18.2+	17.1		
12....	2671.33	113	P	.....	16.7	17.6	.....	17.2		
17....	2676.4	B2	P	.....	17.2	17.8	.....	18.0		
Jan. 4....	2694.3	D118	G	.....	18.2	18.0	.....	19.0+		
5....	2695.3	122	F	.....	18.3	18.0				
8....	2698.4	132	F	.....	18.5	17.8				
Feb. 6....	2727.3	136	P	.....	18.0+	18.0+				
1921	J.D.	Plate	Qy.	26§	83	20				
July 3....	2874.6	B5	P	.....	18.0+	18.0				
Aug. 1....	2903.4	D200	P	.....	17.6	18.0				
1....	2903.45	201	P	.....	17.6	18.0				
Oct. 4....	2967.5	A6	P	19.0	18.5					
Dec. 5....	3029.4	10	F	18.6	18.5+	18.3+				
1922–23	J.D.	Plate	Qy.	26	21	22	84	82		
May 29....	3204.6	B40	P	18.6	18.0					
June 23....	3229.6	A22	P	.....	17.5					
28....	3234.6	S150	F	.....	17.8	.....	19.0+			
July 20....	3256.6	A29	P	18.6	18.5+	.....	17.5			
22....	3258.7	B51	F	.....	.....	.....	17.5			
23....	3259.6	56	F	.....	.....	.....	17.6			
24....	3260.7	S156	P	.....	.....	.....	17.6			
25....	3261.6	163	F	18.6	.....	.....	17.6			
27....	3263.6	D220	P	.....	.....	.....	17.6			
28....	3264.65	222	P	18.6	.....	.....	17.6			
Aug. 20....	3287.6	B58	P	.....	.....	.....	17.7			
22....	3289.6	62	P	.....	.....	.....	17.8			
28....	3295.6	S171	F	18.5	.....	.....	18.0			
Sept. 19....	3317.7	172	F	18.5	.....	.....	19.0+			
24....	3322.6	B67	F	.....	.....	.....	.....	18.5+		
Oct. 16....	3344.4	70	G	.....	.....	.....	.....	17.9		
22....	3350.3	A42	P	.....	.....	.....	.....	18.4		
Jan. 13....	3433.3	B100	F	.....	.....	18.0+	.....	18.5+		
Feb. 15....	3466.3	A47	P	18.5	.....	16.9				
1923–24	J.D.	Plate	Qy.	23	24	25	27	28	29	30
July 16....	3617.8	S254	F	18.0+						
Aug. 7....	3639.8	267	F	17.0						
9....	3641.5	278	P	17.1	18.0+	18.3+				
Oct. 4....	3697.55	H331	VP	17.5+	17.2	17.3				
5....	3698.55	335	G	18.5+	17.3	17.4				
6....	3699.4	S282	VP	.....	17.3	17.4				
8....	3701.35	B142	F	.....	17.4	17.3				
9....	3702.5	A61	G	.....	17.2	17.3				
Nov. 13....	3737.55	H339	G	.....	18.5	18.4	18.5+	19.0+	19.0+	
Dec. 11....	3765.45	348	G	.....	18.5+	19.0	17.2	16.2	17.5	
14....	3768.4	351	P	.....	.....	18.5+	17.5	16.2	18.0	
30....	3784.3	B158	P	.....	.....	.....	18.2	18.2+	18.0	
Jan. 4....	3789.35	H353	P		.....	.....	18.0			
5....	3790.4	S285	G	31	.....	.....	18.2			
6....	3791.4	292	G		.....	.....	18.8	19.0+	19.0	
9....	3794.3	H354	P	18.2+	.....	.....	18.5+	.....	.....	18.5+
31....	3816.3	A69	P	16.8	.....	.....	.....	.....	.....	17.3
Feb. 2....	3818.3	S301	F	17.1	.....	.....	19.0+	.....	19.0+	17.2
3....	3819.3	310	F	17.1	.....	.....	.....	.....	.....	17.2
4....	3820.3	320	F	17.2	.....	.....	.....	.....	.....	17.3
5....	3821.3	H355	G	17.2	.....	.....	.....	.....	.....	17.2
6....	3822.3	357	F	17.2	.....	.....	.....	.....	.....	17.3
7....	3823.3	363	F	17.3	.....	.....	.....	.....	.....	17.2
27....	3843.3	A73	P	18.2+	.....	.....	.....	.....	.....	18.5+

TABLE IV—*Continued*

1924–25	J.D.	Plate	Qy.	32	33	34	35¶	37	40	41	42
July 2....	3969.55	H375	P	17.4							
2....	3969.65	376	F	17.2							
4....	3971.6	S329	P	16.4							
23....	3990.5	336	P	17.5							
24....	3991.6	338	G	18.0							
26....	3993.6	339	G	18.0							
27....	3994.6	344	G	18.0							
28....	3995.5	348	G	17.5							
29....	3996.55	H384	G	17.6							
30....	3997.6	390	G	17.7							
31....	3998.6	395	G	18.0							
Aug. 1....	3999.6	S351	G	18.0							
2....	4000.5	H397	F	18.0							
3....	4001.6	398	P	18.3							
4....	4002.6	404	E	18.0							
5....	4003.5	405	E	18.2							
6....	4004.45	414	E	18.2	18.5+	.....	18.5+				
25....	4023.4	B182	F	18.2	17.6	.....	17.0				
26....	4024.65	D241	F	18.4	17.4	.....	16.8				
27....	4025.63	246	G	18.8+	17.3	18.8+	16.9				
28....	4026.5	S356	G	.....	17.4	.....	16.9				
29....	4027.65	360	F	.....	17.1	18.5+	17.1				
30....	4028.5	H418	G	.....	17.2	17.0	17.0				
31....	4029.45	S361	F	.....	17.3	16.2	17.1				
Sept. 1....	4030.45	362	F	.....	17.3	16.2	17.0				
2....	4031.5	H421	G	.....	17.4	16.8	17.0				
3....	4032.5	426	G	.....	17.3	17.1	17.2				
4....	4033.5	430	G	.....	17.4	16.4	17.0				
20....	4049.4	S360	F	.....	17.5	17.2	17.6				
21....	4050.5	374	F	.....	17.7	17.3	17.7	18.6+			
25....	4054.5	H434	P	.....	18.0	17.3	17.8	16.6			
26....	4055.5	S376	G	.....	.....	17.5	17.8				
27....	4056.5	385	G	.....	18.0	17.6	.....	16.6			
28....	4057.5	388	G	.....	18.0	17.8	18.0	17.2	19.5+		
30....	4059.4	H444	G	.....	18.0	18.0	18.2	17.5	19.0		
Oct. 26....	4085.35	451	G	.....	18.5+	18.6	18.5	.....	19.0		
27....	4086.4	S394	F	.....	.....	18.5	.....	18.5	18.7		
30....	4089.4	401	G	.....	.....	18.6+	18.7+	18.6	18.5		
31....	4090.4	H457	G	.....	.....	.....	.....	18.6			
Nov. 2....	4092.3	464	F	.....	.....	.....	.....	.....	18.5		
18....	4108.4	469	F	.....	.....	.....	.....	.....	18.1		
19....	4109.4	471	G	.....	.....	18.8	.....	18.6			
22....	4112.4	S406	P	.....	.....	.....	.....	.....	18.0		
24....	4114.4	411	G	.....	.....	18.8+	.....	18.8	17.5		
Dec. 28....	4148.35	H507	P	43	44	.....	45	.....	18.4		
30....	4150.35	510	P	.....	.....	.....	.....	.....	18.4	18.5+	18.3+
Jan. 17....	4168.35	S417	P	.....	18.2+	.....	18.3+	.....	18.1	17.1	16.4
19....	4170.3	H514	P	18.2+	16.5	.....	18.2+	.....	18.2	17.4	16.7
26....	4177.35	S424	F	16.8	17.8	.....	17.7	.....	18.3	17.5	17.1
27....	4178.3	433	G	17.0	18.0	.....	17.5	.....	18.4	17.8	17.2
30....	4181.35	H531	P	17.4	18.0	.....	18.0	.....	18.2	18.0	17.4
Feb. 15....	4197.3	S434	P	18.2	18.5+	.....	18.0	.....	18.2	18.2+	18.2+

1925	J.D.	Plate	Qy.	47	48	49	50	51	52	53	54
June 17....	4319.6	H571	F	17.7	17.0	19.0+	19.0+	19.0+	.....	18.6	
19....	4321.6	S483	G	17.5	17.0	.....	.....	.....	.....		
July 16....	4348.55	H576	P	18.5+	18.2+	17.3	17.6	18.2	.....	18.4	
17....	4349.5	580	F	.....	.....	17.2	18.0	18.1	.....	18.4	
19....	4351.6	D251	F	19.0	19.0+	17.2	18.0	18.0	.....		
24....	4356.6	268	G	19.2	19.0+	17.8	17.6	18.3	.....	18.8	
26....	4358.5	S496	G	19.0+	.....	18.0	18.0	18.7+	18.6+		
Aug. 14....	4377.45	H586	F	18.3	.....	19.0	19.0	.....	17.3	18.6	
16....	4379.7	598	P	18.0	.....	19.2	19.0+	.....	17.1	18.6	18.8+
Sept. 11....	4405.7	604	VP	18.2	.....	19.0	.....	.....	18.0	.....	17.3
12....	4406.6	607	VP	18.6	.....	18.8+	.....	.....	18.5	18.5	15.3
19....	4413.6	S499	P	.....	.....	.....	.....	.....	18.5	18.5	15.7
20....	4414.7	H609	F	19.0	.....	19.2	.....	.....	18.5	18.8	15.7
Oct. 9....	4433.3	615	P	.....	.....	.....	.....	.....	18.8+	18.8	18.8+
17....	4441.45	S505	F	19.0+	.....	19.0+	.....	.....	.....	.....	19.0
20....	4444.35	H617	F	.....	.....	.....	.....	.....	19.0	18.2	19.0+
Nov. 10....	4465.35	620	P	.....	.....	.....	.....	.....	.....	18.2	
20....	4475.3	621	P	.....	.....	.....	.....	.....	.....	18.2	
Dec. 8....	4493.35	H624	G	.....	.....	.....	.....	.....	.....	18.8	

TABLE IV—*Continued*

1925–26	J.D.	Plate	Qy.	55	56	57	58	59	60	61	
June 17....	4319.6	H571	F								
19....	4321.6	S483	G	19.0+							
July 16....	4348.55	H576	P	17.3							
17....	4349.5	580	F	17.3							
19....	4351.6	D251	F	17.6							
24....	4356.6	268	G	18.0							
26....	4358.5	S496	G	18.1							
Aug. 14....	4377.45	H586	F	18.8							
16....	4379.7	598	P	18.8	19.0+	.....	19.0+				
Sept. 11....	4405.7	604	VP	18.5+	.....	.....	18.5				
12....	4406.6	607	VP	.....	18.5+	.....	18.5				
19....	4413.6	S499	P	.....	17.8						
20....	4414.7	H609	F	.....	17.7	18.5+	18.4				
Oct. 9....	4433.3	615	P	.....	18.8	17.4	18.6				
17....	4441.45	S505	F	.....	19.0	17.8	18.6	19.0+			
20....	4444.35	H617	F	.....	19.0+	17.7	17.5				
Nov. 10....	4465.35	620	P	.....	.....	18.5	18.5	16.5	18.8+		
20....	4475.3	621	P	.....	.....	18.5+	18.6	17.5	16.6		
21....	4476.5	Se6276	F	.....	.....	.....	.....	17.5	16.5		
Dec. 8....	4493.35	H624	G	.....	.....	.....	18.6	18.3	17.8	18.6+	
10....	4495.35	626	P	.....	.....	.....	.....	18.4			
Jan. 6....	4522.3	627	P	.....	.....	.....	.....	18.8+	18.8+	16.8	
14....	4530.3	SS 45	F	.....	.....	.....	.....	.....	.....	17.3	
15....	4531.3	H629	P	.....	.....	.....	.....	.....	.....	17.5	
Feb. 16....	4563.3	634	P	.....	.....	.....	.....	.....	.....	18.0+	

1926–27	J.D.	Plate	Qy.	62	63	64	65	66	67	68	69
June 4....	4671.65	S533	P	17.7							
13....	4680.6	D304	VP	17.4	.....	.....	.....	.....	18.3+		
14....	4681.6	309	VP	17.4	18.0+	18.0+	.....	.....	18.0+		
July 3....	4700.6	315	G	17.8	17.3	17.5	18.8+	.....	18.2		
12....	4709.55	H668	F	18.2	17.4	18.0	18.2	19.0+	18.2		
14....	4711.6	S539	G	18.4	17.4	18.0	18.3	.....	18.3		
Aug. 11....	4739.5	540	F	19.0+	17.7	18.0	18.2+	18.2	18.3		
Sept. 3....	4762.4	H675	E	.....	18.8	18.5	.....	19.0	17.5		
5....	4764.35	686	E	.....	18.8	18.8	.....	19.2	17.4		
11....	4770.4	690	E	.....	18.8	18.8	.....	19.2	17.4		
13....	4772.4	703	E	.....	19.0+	19.0+	.....	19.2	18.2	19.2+	
Oct. 4....	4793.3	S545	G	.....	.....	.....	.....	19.2	18.2	17.7	
11....	4800.3	H708	E	.....	.....	.....	.....	19.3+	18.5	18.0	
12....	4801.3	S547	F	.....	.....	.....	.....	.....	18.4	18.1	19.0+
Nov. 2....	4822.5	BB104	G	.....	.....	.....	.....	.....	18.5	18.4	17.4
3....	4823.5	117	G				.....	.....	18.5	18.6	17.4
Dec. 1....	4851.5	150	P	70	71	72	.....	.....	18.2	18.5	18.2
Jan. 1....	4882.3	166	F				.....	.....	18.3	19.0+	18.6
2....	4883.3	180	F	19.0+	.....	.....	.....	.....	18.4+	.....	18.7
6....	4887.3	197	P	18.5	.....	.....	.....	.....	17.7+		
27....	4908.35	H715	G	18.1	19.0+	.....	.....	.....	17.7	.....	19.0
Feb. 6....	4918.35	720	F	18.6	16.6	19.0+	.....	.....	17.5	.....	19.3
25....	4937.3	730	P	18.6	18.1	17.0	.....	.....	18.3	.....	19.2+
26....	4938.3	732	P	18.6+	18.4	17.3	.....	.....	18.3		

TABLE IV—*Continued*

1927	J.D.	Plate	Qy.	73	74	75	76	77	78	79	
May 23....	5024.65	H770	P	17.4	17.6+						
June 3....	5035.6	780	F	17.6	18.0						
4....	5036.6	785	G	17.7	17.7	18.5+					
20....	5052.6	789	P	17.7	17.8	18.0+					
July 1....	5063.6	798	G	18.0	17.8	18.4					
21....	5083.5	812	F	18.8	18.4	18.2					
29....	5091.5	815	P	18.2+	18.5+	18.0	18.0+				
Aug. 22....	5115.45	824	G	18.6+	.....	18.5	17.1				
23....	5116.45	828	G	.....	.....	18.3	17.2				
31....	5124.4	832	F	.....	.....	18.3	17.5	19.0+	19.0+		
Sept. 11....	5145.35	S577	F	.....	.....	18.8	18.0	17.7	18.2		
23....	5147.6	H848	F	.....	.....	19.0	18.5	17.8	18.2		
25....	5149.6	855	F	.....	.....	19.0	18.5	17.8	18.3		
Oct. 22....	5176.35	S591	F	.....	.....	18.8+	18.5+	19.0	19.0+		
Nov. 1....	5186.3	H866	P	.....	.....	.....	.....	18.8	.....	18.5+	
23....	5208.5	S603	G	.....	.....	.....	.....	19.0+	.....	16.7	
26....	5211.3	H867	G	.....	.....	.....	.....	.....	.....	16.8	
28....	5213.45	885	F	.....	.....	.....	.....	.....	.....	16.6	
Dec. 19....	5234.4	888	F	.....	.....	.....	.....	.....	.....	17.5	

* Add 2,410,000.

† Add 2,420,000.

‡ A plus sign following the magnitude indicates limiting magnitudes of plates on which the novae did not appear. Novae are tabulated according to observing seasons, but numbered serially according to dates of discovery. High serial numbers in the earlier seasons represent novae found on re-examining the older plates.

§ Number 26 appeared in October, 1921, and by December had risen to 18.6. During the season 1922–1923 it remained about constant at 18.6, and from July, 1923, to December, 1925, at about 18.0. In January, 1925, it began to fade, but declined so slowly that it did not reach 19.5 until January, 1927. In 1928 it did not appear even on the best plates.

¶ Numbers 36, 39, and 46 are not listed. Number 36 first appeared in August, 1917, after an unobserved interval of two years, and has since remained constant at about 18.2. Number 39 first appeared in February, 1924, and is still visible, varying in an apparently irregular manner between 19.0 and 19.5. Number 46 appeared in Region 4, about 42″ from the nucleus, on the first plate of the season 1926–1927. The data are as follows:

J.D. 2,424,709.6	18.7
2,424,731.5	18.8
2,424,739.5	19.2
2,424,762.5	19.4
2,424,800.4	19.7+

On three plates between the last two dates, with limiting magnitudes about 19.4, the nova was invisible.

seen are included in the table. Magnitudes of stars in the immediate region of the nucleus fainter than about 18.0 are uncertain because of the nebulous background, but outside that region they should be fairly reliable down to about 19.0.

The significant data are collected in Table V. These include the observed duration of a nova; the unobserved intervals immediately preceding and following the observations; the magnitudes at the first observation, at maximum, and at the last observation; the intervals between the first observation and maximum, and between maximum and the next following observation; the number of observations; the approximate co-ordinates measured from the nucleus along the major axis (plus to the south preceding) and the minor axis (plus to the north preceding); and, finally, the latter co-ordinates multiplied

TABLE V

NOVAE IN M 31

Nova	Duration*			Magnitude				Interval between Observations		No. Obs.	Position†		
	Prec.	Obs.	Fol.	First Obs.	Last Obs.	Max. Obs.	Max. Cal.	1st – Max.	Max. – N'xt		X	Y	$4Y$
2.....	4hr	56d	....	17.6	18.0	16.7	16.7	2.8d	1d	9	+ 4.0	+ 1.0	+ 4.0
3.....	4hr	56	....	17.6	17.6	17.1	17.1	3.8	25	9	+ 2.5	+ 2.2	+ 8.8
4.....	21d	35	29	17.5	18.6	17.5	16.7	0	6	3	−10.1	− 0.0	− 0.1
5.....	26	32	28	17.3	18.2	17.3	16.4	0	3	3	+ 3.9	− 2.2	− 8.8
6.....	29	62	25	16.5	18.3	16.5	15.5	0	28	4	− 5.2	+ 0.6	+ 2.4
7.....	33	2	23	17.1	17.1	17.1	15.9	0	2	2	+ 1.5	+ 1.95	+ 7.8
8.....	25	1	....	17.5	17.4	17.4	16.6	1	....	2	+ 1.8	+ 1.1	+ 4.4
9.....	25	1	....	17.5	17.5	17.5	16.7	0	1	2	− 8.8	− 2.7	−10.8
10.....	38	26	63	17.4	17.8	17.4	16.1	0	1	5	− 7.0	+ 2.9	+11.6
11.....	38	1	25	17.6	17.7	17.6	16.3	0	1	3	+ 4.9	− 3.5	−14.0
12.....	63	30	....	17.0	17.5	17.0	.....	0	1	5	− 2.2	+ 3.3	+13.2
13.....	63	30	....	17.2	18.5	17.2	.....	0	1	5	+ 5.7	+ 0.1	+ 0.4
14.....	21	8	26	16.1	16.3	16.1	15.3	0	2	4	− 0.1	− 5.3	−21.2
15.....	.....	176	....	17.7	18.2	17.4	.....	23	2	17	− 0.8	+ 3.7	+14.8
16.....	26	28	25	17.1	17.4	17.1	16.2	0	4	5	+ 4.15	+ 0.6	+ 2.4
17.....	14	1	30	15.8	16.6	15.8	15.2	0	1	2	− 2.2	+ 1.4	+ 5.6
18.....	25	4	29	17.2	17.4	17.2	16.3	0	1	3	− 0.2	+ 1.3	+ 5.2
19.....	27	29	29	16.0	18.5	15.9	15.0	1hr	2	8	+ 2.3	− 1.9	− 7.6
20.....	27	29	....	17.7	17.8	17.6	16.7	2	5	9	− 2.6	− 0.8	− 3.2
21.....	.....	29	22	18.0	17.8	17.5	16.7	24	5	3	− 4.65	− 1.5	− 6.0
22.....	33	.....	....	16.9	16.9	16.9	15.8	.....	....	1	− 1.6	− 0.1	− 0.4
23.....	22	2	56	17.0	17.1	17.0	16.2	0	2	2	+ 1.8	+ 2.65	+10.6
24.....	56	40	28	17.2	18.5	17.2	.....	0	1	6	+ 9.6	+ 1.25	+ 5.0
25.....	56	68	3	17.3	19.0	17.3	.....	0	1	7	+12.35	+ 0.5	+ 2.0
26.....	64	5.5yr	....	19.0	19.5	18.0	.....	3yr	....	....	+ 3.2	− 5.4	−21.6
27.....	28	26	27	17.2	18.8	17.2	16.3	0	3	6	− 0.5	+11.7	+46.8
28.....	28	3	16	16.2	16.2	16.2	15.3	0	3	2	− 5.45	− 4.05	−16.2
29.....	28	26	27	17.5	19.0	17.5	16.6	0	3	4	+ 6.8	− 3.5	−14.0
30.....	22	7	20	17.3	17.2	17.2	16.4	2	1	7	+11.0	− 6.25	−25.0
31.....	22	7	20	16.8	17.3	16.8	16.0	0	2	7	0.0	− 2.7	−10.8
32.....	.....	55	1	17.4	18.4	16.4	16.4	2	19	19	+ 6.8	− 0.6	− 2.4
33.....	19	36	26	17.6	18.0	17.1	16.4	4	1	17	+ 0.75	−2.95	−11.8
34.....	1	58	3	17.0	18.5	16.2	16.2	1	1	15	− 0.45	− 3.6	−14.4
35.....	19	62	4	17.0	18.5	16.8	16.3	1	1	18	+ 2.1	+ 1.5	+ 6.0
36‡....	.....	11yr	....	18.2	18.2	.....	.....	.....	.....	....	−10.25	+ 2.6	+10.4
37.....	4	60	34	16.6	18.8	16.6	16.4	0	2	9	−32.4	−14.3	−57.2
38.....	.....	31	25	18.7	19.4	18.7	.....	0	1	8	+16.3	− 9.9	−39.6
39§....	25	4.5yr	....	19.0	19.5	.....	.....	.....	.....	....	+ 0.7	+ 6.65	+26.0
40.....	2	138	....	19.0	18.2	17.5	17.5	55	34	16	+ 4.35	−10.3	−41.2
41.....	18	13	16	17.1	18.0	17.1	16.4	0	2	5	− 5.0	+ 1.3	+ 5.2
42.....	18	13	16	16.4	17.4	16.4	15.7	0	2	5	+ 3.8	+ 1.3	+ 5.2
43.....	7	20	....	16.8	18.2	16.8	16.5	0	1	4	+ 4.4	− 0.15	− 0.6
44.....	2	11	16	16.5	18.0	16.5	16.4	0	7	4	+ 7.9	−11.9	−47.6
45.....	7	20	....	17.7	18.0	17.5	17.3	1	3	4	+ 3.05	−12.25	−49.0
46.....	.....	53	8	18.7	19.4	18.7	.....	0	22	4	+41.5	+ 3.7	+14.8
47.....	.....	95	27	17.7	19.0	17.5	.....	2	27	11	− 8.2	− 0.1	− 0.4
48.....	.....	2	27	17.0	17.0	17.0	.....	0	2	2	+18.4	+ 2.7	+10.8
49.....	29	66	27	17.3	19.2	17.2	16.2	1	2	9	− 9.6	− 7.5	−30.0
50.....	29	29	2	17.6	19.0	17.6	16.6	0	1	6	−13.65	+ 1.1	+ 4.4
51.....	29	8	2	18.2	18.3	18.0	17.1	3	5	4	+10.2	− 0.5	− 2.0

TABLE V—*Continued*

Nova	Duration*			Magnitude				Interval Between Observations		No. Obs.	Position†		
	Prec.	Obs.	Fol.	First Obs.	Last Obs.	Max. Obs.	Max. Cal.	1st – Max.	Max. – N'xt		*X*	*Y*	4*Y*
52.....	19d	67d	19d	17.3	19.0	17.1	16.4	2	26	8	+ 3.15	− 0.9	− 3.6
53.....	.....	174	....	18.6	18.8	18.2	.....	126	21	15	− 2.8	+ 0.2	+ 0.8
54.....	26	36	3	17.3	19.0	15.3	15.3	0.8	7	6	−18.6	+ 1.65	+ 6.6
55.....	27	31	26	17.3	18.8	17.3	16.4	0	1	7	+ 5.2	− 0.65	− 2.6
56.....	7	28	3	17.8	19.0	17.0	16.4	1	19	4	− 1.1	+14.5	+58.0
57.....	19	32	10	17.4	18.5	17.4	16.7	0	8	4	+ 4.6	+ 0.7	+ 2.8
58.....	26	88	....	18.5	18.6	17.5	17.5	39	21	9	− 1.6	− 1.8	− 7.2
59.....	24	30	27	16.5	18.4	16.5	15.7	0	10	5	+11.4	− 1.6	− 6.4
60.....	10	18	29	16.6	17.8	16.5	16.1	1	17	3	−24.3	− 7.5	−30.0
61.....	29	9	32	16.8	17.5	16.8	15.8	0	8	3	+ 1.9	− 0.3	− 1.2
62.....	.....	40	28	17.7	18.4	17.4	16.7	9	1	6	+17.5	+ 2.2	+ 8.8
63.....	19	70	2	17.3	18.8	17.3	16.6	0	9	7	+ 1.75	− 1.1	− 4.4
64.....	19	70	2	17.5	18.8	17.5	16.8	0	9	7	+ 0.8	+ 1.75	+ 7.0
65.....	9	2	28	18.2	18.3	18.2	17.8	0	2	2	− 0.9	− 1.2	− 4.8
66.....	30	54	7	18.2	19.2	18.2	17.2	0	23	6	+ 2.9	− 6.0	−24.0
67.....	19	238	....	18.2	18.3	17.4	16.6	64	6	21	+ 4.1	+ 1.5	+ 6.0
68.....	21	58	31	17.7	18.5	17.7	16.9	0	7	6	−10.9	+ 2.4	+ 9.6
69.....	21	96	19	17.4	19.3	17.4	16.6	0	1	7	+ 7.8	− 3.5	−14.0
70.....	4	50	1	18.5	18.6	18.1	16.8	21	10	4	−18.1	+ 0.1	+ 0.4
71.....	10	20	....	16.6	18.4	16.6	16.1	0	19	3	−16.9	− 0.5	− 2.0
72.....	19	1	....	17.0	17.3	17.0	16.2	0	1	2	+ 2.5	− 1.3	− 5.2
73.....	.....	59	8	17.4	18.8	17.4	.....	0	11	6	+ 4.3	− 0.35	− 1.4
74.....	11	48	8	18.0	18.4	17.7	17.7	1	16	5	+16.1	− 2.6	−10.4
75.....	11	86	27	18.4	19.0	18.0	16.9	28	24	9	− 5.95	− 2.6	−10.4
76.....	24	34	27	17.1	18.5	17.1	16.3	0	1	6	+ 3.45	− 0.6	− 2.4
77.....	21	41	22	17.7	18.2	17.7	16.9	0	2	5	+ 6.05	+ 7.55	+30.2
78.....	21	4	27	18.8	18.3	18.2	17.4	0	2	3	+14.5	+ 0.5	+ 2.0
79.....	22	26	....	16.7	17.5	16.7	15.9	0	3	4	+ 1.5	+ 2.0	+ 8.0
80.....	17	124	2	18.0	18.6	17.5	17.3	35	1	14	+ 3.0	+ 2.8	+11.2
81.....	27	7	18	16.7	18.0	16.7	15.8	0	2	5	− 7.4	−11.7	−46.8
82.....	22	6	....	17.9	18.4	17.9	17.1	0	6	2	− 7.1	+ 1.8	+ 7.2
83.....	27	64	62	17.6	18.5	17.6	16.7	0	64	3	+ 3.0	− 2.35	− 9.4
84.....	22	39	22	17.5	18.0	17.5	16.7	0	2	10	+12.2	− 1.15	− 4.6
85.....	28	1	33	17.4	17.4	17.4	16.5	0	1	2	−11.5	− 8.3	−25.2
86.....	20	0	25	16.7	.....	16.7	16.0	.....	....	1	− 9.4	− 3.3	−13.2
Mean¶	.....	.....	....	.....	.....	17.20	16.43	.....	....	....	.....	.....	.....

* Under "Duration" are listed the length of the unobserved interval preceding the first observation, the interval between the first and the last plates on which the nova appeared, and the unobserved interval following the last observation. When no preceding interval is given, the nova appeared on the first plate of an observing season. The intervals are expressed in days unless otherwise designated.

† Positions are given with respect to the nucleus, *X* along the major axis (plus to the south preceding), *Y* along the minor axis (plus to the north preceding). The last column gives the *Y* co-ordinate corrected for foreshortening due to the inclination of the nebula to the line of sight.

‡ Number 36 first appeared in August, 1917, following an unobserved interval of two years, and has maintained a constant luminosity, 18.2, up to the present time.

§ Number 39 first appeared in February, 1924, and has remained visible up to the present, varying between 19.0 and 19.5.

¶ The mean observed maximum represents 82 novae; Nos. 26, 36, and 39 are rejected. The mean calculated maximum represents 71 novae. In addition to Nos. 26, 36, and 39, the following are omitted on the grounds that the preceding unobserved intervals are excessive—Nos. 12, 13, 15, 24, 25, 38, 46, 47, 48, 53, and 73.

by 4 in order to correct for the inclination of the plane of the spiral to the line of sight. The position of the major axis was estimated on several plates of long exposures on different scales. The line selected was then drawn on a print of the nebula and the orientation was measured with reference to stars whose positions were accurately known. The position angle of the major axis as thus determined is N 36°.7 E. The ratio of the axes was estimated on long exposures as about 4 to 1, hence the inclination of the spiral to the line of sight is of the order of 15°. The nuclear region, by analogy with other spirals of the same type which are seen edge on, is probably much less flattened than the nebula as a whole—a circumstance to be considered in reconstructing the image as it would appear were the plane perpendicular to the line of sight.

An inspection of Table V indicates the necessity of some culling before entering upon a general discussion. The most conspicuous abnormalities occur in Nos. 26, 36, and 39. Number 26 was first observed in October, 1921, at about 19.0. It brightened slowly until by July, 1924, it was about 18.0. By December of the same year it had begun to fade and now, four years later, it has vanished. The description appears to be that of a long-period or an irregular variable rather than of a nova, and analogies may possibly be found in variables Nos. 15 and 19 in M 31 and Nos. 1 and 2 in M 33. Numbers 36 and 39 appeared, the former between 1915 and 1917, the latter in February, 1924, at about 18.6 and 19.0, respectively, and have maintained constant luminosities up to the present date. These stars may be of the same general type as No. 26, or possibly they may be ordinary stars which have drifted out from behind clouds of dark nebulosity. Variable No. 20 in M 31 may be a similar object. At any rate, the three stars clearly are not normal novae and can be discarded in statistical discussions. It is significant also that they are among the faintest which were originally observed as novae.

Number 22, discovered by Humason in February, 1923, appeared on only one plate. The image, however, shares the distortion, due to astigmatism in the mirror, which the other stars exhibit, hence there is no doubt as to the reality of the nova. Number 86 is a similar case, but no other object is included which has not appeared on more than one plate. For the following sixteen, however, the observations

are very meager: Nos. 7, 8, 9, 11, 17, 18, 23, 28, 48, 51, 61, 65, 71, 72, 82, and 85. In these cases there are either but two observations or three or four covering a short interval of time.

Numbers 15, 38, 46, 47, 48, 53, 62, and 73 appeared on the first plates obtained after unobserved intervals of more than a hundred days. The maximum observed magnitudes of some of these are probably much fainter than the real maxima, which may have occurred some weeks earlier. This offers a ready explanation of the faintest two maxima listed in Table V, 18.7 for Nos. 38 and 46. In eight other cases, the first observations follow unobserved intervals of more than thirty days.

In five cases, Nos. 40, 58, 67, 75, and 80, observed maxima occurred several weeks after the first observations, although it is uncertain whether these represent primary or secondary maxima. In the discussions which follow, all novae are included excepting Nos. 1, 26, 36, and 39. The twenty-three novae listed in the two preceding paragraphs, however, and, to a lesser degree, the six mentioned at the beginning of the present paragraph are entitled to smaller weight than the remaining novae.

These stars have been called novae because they flare up very suddenly, fade slowly, and vanish completely. In the eighty-five cases listed, none has reappeared during the eighteen years covered by the observations at Mount Wilson. Occasional plates at other observatories extend the period less certainly over an additional ten years. Novae are the only galactic objects which exhibit such characteristics.

These stars at maxima, moreover, are about the brightest objects in the spiral, and average more than 2 mag. brighter than the Cepheids. In the case of No. 54, a small-scale slitless spectrogram was obtained by Humason, at the Newtonian focus of the 100-inch reflector, some six days after maximum. The continuous spectrum extended well into the violet, and faint patches of emission could be seen in the region of the Balmer lines of hydrogen. The color-index was less than the errors of the determination, which were estimated as about 0.2 mag. These indications, while by no means conclusive, strengthen the analogy with galactic novae.

LIGHT-CURVES OF NOVAE

The data on individual stars are usually fragmentary, but in a few cases they are sufficient to indicate the general form of the light-curves. The sharp initial rise, familiar in galactic novae, appears to be certainly present. The most conspicuous examples are novae Nos. 2 and 3, which Ritchey found on a plate exposed on the morning of September 13, 1909, although they were not visible on a plate exposed four hours earlier. Both novae had brightened at least 0.7 mag. within that interval. By the next exposure, on the late evening

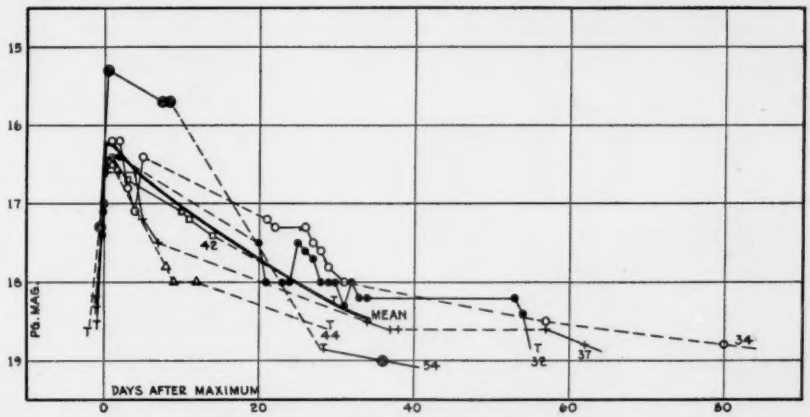


FIG. 5.—Light-curves of six novae in M 31 observed near maximum

of the same day, No. 2 had brightened another 0.7 mag. Nova No. 34 rose 1.5 mag. above the limits of the plates during an unobserved interval of about twenty hours, and on the next plate, a day later, was 0.8 mag. brighter yet. Other instances are Nos. 37 and 44, which appeared about 2 mag. above the limits of the plates after unobserved intervals of four and two days, respectively. Number 54 first appeared at 17.3 after a long unobserved interval, but on the next plate, twenty-two hours later, it was 15.3. Since these data set lower limits only, the actual rates of brightening may be considered comparable with those of galactic novae.

Data for six novae well observed near maximum, Nos. 32, 34, 37, 42, 44, and 54, furnish a mean light-curve, shown in Figure 5, in which the rapid rise, the minor fluctuations, and the slow decline

are conspicuous. A composite light-curve, incorporating most of the data in Table IV, has also been constructed on the assumption that the actual maxima of the various novae occurred in the middle of the unobserved interval preceding the first observation. Novae for which this unobserved interval exceeds thirty days were excluded, and, in addition, those for which the observed maxima occurred long after the first observations, and Nos. 21 and 70, for which the data were so poorly distributed that the curves could not be reconstructed with any certainty. This selection reduced the number of available novae to fifty-seven.

Mean magnitudes were computed for various dates after maximum, and the mean curve thus indicated is approximately linear with a slope of about 0.05 mag. per day. The region of greatest weight, however, is between ten and twenty days after maximum, the earlier and later portions depending largely on linear extrapolations or interpolations from rather meager data. A comparison of the mean magnitudes on the two curves, the composite curve, and that for the six well-observed novae indicates that the latter is somewhat steeper, but that in the region where the former has greatest weight, the two agree very closely. This suggests that the six selected novae are representative cases, and that their behavior near maximum is probably typical.

The composite curve indicates a mean maximum of about 16.6 as against 16.2 for the selected curves. The discrepancy is due to two factors—the inclusion of fainter novae in the composite curve and the linear extrapolations backward from the first observations. The latter is the more serious since it ignores the humps at maxima which are indicated by the well-observed novae. The point is rather important in connection with the frequency distribution of maxima; hence some form of the mean curve in this region must be adopted, even though it be somewhat arbitrary. Fortunately, the limits of the selection are not widely separated, and a simple mean between the two curves will not be grossly in error. With this procedure, the hypothetical mean maximum is 16.4 and the mean magnitudes at three, five, seven, and ten days after maximum are 16.7, 16.8, 16.9, and 17.1, respectively. At ten days after maximum the two curves are in agreement, and from then on the composite curve may be used with its linear slope of about 0.05 mag. per day.

FREQUENCY DISTRIBUTION OF MAXIMA

The maxima of the various novae can be determined with the aid of this standard light-curve by extrapolating backward from the first observations, on the assumption that the maxima occurred midway in the unobserved intervals preceding these observations and that the various individual curves are approximately parallel. In a few cases, departures from the formal procedure, clearly indicated by the data, have led to more probable values for the maxima. The

TABLE VI

MEAN MAGNITUDES ON THE COMPOSITE LIGHT-CURVE AND THE LIGHT-CURVE FOR SIX WELL-OBSERVED NOVAE

Interval in Days after Max.	Composite	Six Novae	Mean
0	(16.6)	16.23	(16.4)
1	.....	16.25	(16.5)
3	16.8	16.5	(16.7)
5	16.95	16.65	(16.8)
7	17.0	16.8	(16.9)
10	17.14	17.05	(17.1)
14	17.33	17.3	.....
15	17.37	17.4	.....
20	17.63	17.7	.....
25	17.88	18.0	.....
30	18.1	(18.3)	.....
40	18.6	(18.6)	.....

maxima, observed or hypothetical, are listed in the eighth column of Table V, and exceptional cases are given in the footnotes.

A frequency-curve of seventy-one maxima[1] is shown in Figure 6. In order to smooth the curve somewhat, the numbers of novae have also been summed over intervals of 0.3 mag., centered upon successive tenths. The restricted range, the symmetrical form, and the narrow maximum are unexpected features; but they appear to be definite characteristics, independent of the particular method of analysis. Any reasonable treatment of the data would lead to results of the same general character. The mean magnitude is 16.43, and the most frequent magnitude, about 16.5.

[1] In addition to Nos. 26, 36, and 39, novae Nos. 12, 13, 15, 24, 25, 38, 46, 47, 48, 53, and 73 are omitted because the unobserved intervals preceding the first observations exceed forty days.

Although the general character of the frequency-curve appears to be established, the details depend upon extrapolations of a somewhat arbitrary nature. For this reason a similar curve has been constructed, showing the frequencies of magnitudes fourteen days after maximum, where the extrapolations and interpolations are reduced to a minimum. This curve (see Fig. 7) is of the same general character as that in Figure 6. The mean magnitude is 17.37, and the

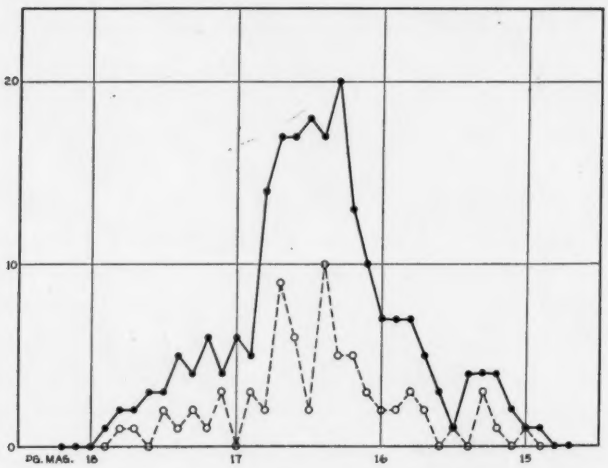


FIG. 6.—Frequency distribution of magnitudes at maxima among 71 novae in M 31. Open circles indicate numbers for each 0.1 mag.; black disks indicate sums over intervals of 0.3 mag., centered on successive tenths. The maxima, in general, have been derived from the mean light-curve of novae in M 31, on the assumption that maxima occurred at the middle of the unobserved intervals preceding the first actual observations of the novae.

most frequent magnitude is about 17.35. The mean magnitude, extrapolated according to the standard curve, gives a maximum of 16.44, in agreement with Figure 6, hence the adopted maximum, 16.5, appears to be a conservative estimate.

These frequency-curves indicate that the novae observed in M 31 are very similar objects. Their maxima, for instance, can be predicted with a probable error of the order of 0.5 mag., and the total range is of the order of 3 or 4 mag. The data indicate only two maxima fainter than 17.5, although the limits of the observations are from 1.0 to 2.5 mag. fainter. It is probable that some faint novae

have been missed, especially in the very dense nuclear region; but the wealth of material covering other regions clearly indicates that the number is relatively small. Variables were frequently found whose maxima were around 19.0 and were followed to 20.0 and fainter, but no very faint novae were found except on plates following long unobserved intervals. The relative distribution of maximum magnitudes actually observed among novae and variables is shown in Figure 8.

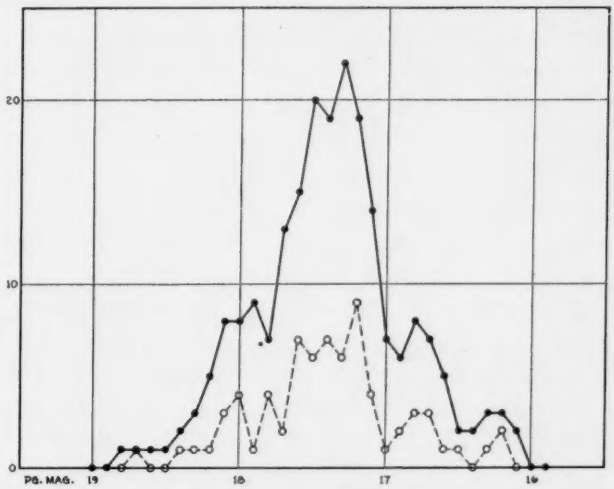


FIG. 7.—Frequency distribution of magnitudes of novae 14 days after maxima. Open circles indicate numbers for each 0.1 mag; black disks indicate sums over intervals of 0.3 mag., centered on successive tenths.

At the distance indicated by the Cepheids, the most frequent magnitude at maximum, 16.5, corresponds to an absolute photographic magnitude of -5.7. Since the color-indices of novae at maximum are negligible, the visual magnitude is about the same. Absolute magnitudes at maximum are reliably determined for only two galactic novae—Nova Persei (1901) at -5.0 and Nova Aquilae (1918) at -9.2. The various discussions of the mean magnitude of galactic novae at maxima, mainly by Lundmark, have led to values ranging from -3 to -9, the last published value[1] being -6.1. This

[1] "Studies of Anagalactic Nebulae," *Meddelanden från Astronomiska Observatorium, Upsala*, No. 30, 1927.

agrees as closely as can be expected with the value for novae in M 31 and suggests no considerable revision of Shapley's zero point in the period-luminosity curve for Cepheids.

Aside from the possible uncertainty in this zero point, the absolute magnitudes of novae in M 31 are determined with a much greater precision than those in the galactic system. Hence, if the novae in the two systems are objects of the same sort, the frequency-curves for those in M 31 may be used to advantage in studying the distribution of novae in the galactic system. The approximate agreement in

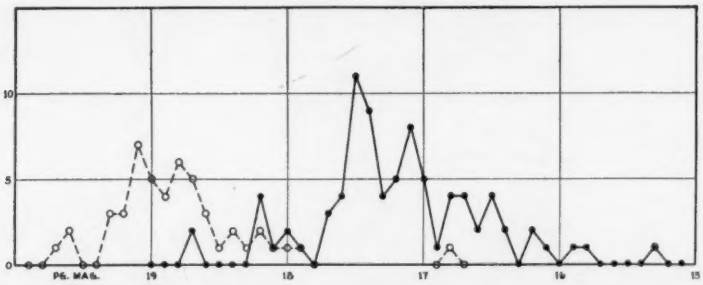


FIG. 8.—Frequency distribution of observed maxima of novae and variable stars in M 31. Black disks refer to novae; open circles, to variable stars. The points indicate numbers for each 0.1 mag. The diagram emphasizes the *completeness* of the data for novae, and hence the reality of the restricted range in magnitudes at maxima indicated by Figures 6 and 7.

the mean magnitudes at maximum, the characteristics of the light-curves, and the fragmentary evidence of color and spectrum suggest very strongly that the two groups actually are similar. The range in magnitude at maximum among galactic novae is unknown at present, and no definite conclusion can be formulated as to its value until the number of reliable individual distances is sufficient to permit a statistical treatment of the data. If the use of the frequency distribution of maxima in M 31 should lead to wholly inconsistent results for the distances of galactic novae, the question of similarity would be definitely settled in the negative.

Four novae have been found thus far in M 33, but each has first appeared on plates following unobserved intervals of several months. Two of these are discussed in an earlier paper, and data for the two

later objects are given in the footnote below.[1] All four were obviously on the descending branch of their light-curves when first observed, hence no certain information is available concerning their maxima. The first observations give 17.2, 17.9, 18.1, and 17.9 mag. The mean, 17.7, is about the same as the mean of the ten similar cases in M 31, namely, 17.6, and leaves little doubt that the novae in the two spirals are similar objects. This evidence, weak as it is, materially strengthens the analogy between the spirals and the galactic system based upon the observations of Cepheids.

The one nova thus far observed in the Magellanic Clouds[2] reached an absolute magnitude of at least -5.0, but, as far as can be judged from the records, probably did not exceed -7.0.

The occasional novae which have appeared in the smaller extragalactic nebulae and which have attained luminosities that are respectable fractions of the total luminosities of the nebulae themselves are clearly a different sort of object from the normal novae in M 31 and 33. They are generally classed with S Andromedae, the nova of 1885 in M 31, as a rare and peculiar type.

FREQUENCY OF NOVAE

The numbers of novae for the various observing seasons are given in Table VII. The more systematic spacing of the plates accounts for the larger numbers of novae discovered during the last five seasons,

[1] *Mt. Wilson Contr.*, No. 310; *Astrophysical Journal*, 63, 236, 1926. Nova No. 3 appeared 6′.6 south following the nucleus of M 33; No. 4 appeared 1′.9 south and 0′.4 following variable No. 34. The magnitudes are as follows:

Plate	J.D.	No. 3	No. 4
H619	2,424,444.5	.....	.....
623	2,424,493.3	18.1	
673	2,424,610.6	18.6	
621	2,424,918.3		
817	2,425,091.6	.....	17.7
834	2,425,124.5	.....	18.1
847	2,425,147.6	.....	19.2
868	2,425,211.3		

After the present paper was prepared for publication, W. Baade announced the discovery on October 15, 1928, of a nova in M 33, which was observed at the sixteenth magnitude. This fifth nova strengthens the analogy with M 31 in a very evident manner.

[2] *Harvard College Observatory Bulletin*, Nos. 847 and 851, 1927.

hence these data may be regarded as representative. Comparison of the numbers of novae with the months covered by the observations indicates a frequency of sixteen or seventeen novae per year. This, however, neglects the very considerable gaps in the observations. By assuming that each plate shows all novae which appeared during the preceding ten days, the average number per year is found to be about thirty. If the preceding interval is put at fifteen days, the annual average is 21.5; for an interval of twenty days, the average is 18.5. Since the ten-day interval is almost certainly too short and the twenty-day interval is probably too long, the intermediate value may be accepted as a reasonable approximation.

TABLE VII

FREQUENCY OF OBSERVED NOVAE

Season	Novae	Interval	Season	Novae	Interval
1909–1910.....	3	3.0 mo.	1923–1924.....	8	7.5 mo.
1917–1918.....	7	6.0	1924–1925.....	11	8.0
1918–1919.....	4	7.0	1925–1926.....	15	8.0
1919–1920.....	5	7.0	1926–1927.....	12	8.5
1920–1921.....	5	7.0	1927——.....	7	7.0
1921–1922.....	1	7.0			
1922–1923.....	4	8.5			
Total.....	29	45.5	Total.....	53	39.0

The resulting annual average of 21.5 must be increased to include novae which have been missed in the study of the plates, those which have been below the limits of plates made under exceptionally poor conditions, and still others which have appeared in regions not covered by the plates. The first and last of these sources of error are believed to be of minor importance. Thus, during the re-examination of the entire series, eight additional novae were found on plates prior to the season 1923–1924, but none on the more recent plates. In the outer region of the nebula, novae appear to be very rare; in Region 4, 48′ from the nucleus, only one example has been found on the sixty-five plates covering four observing seasons. Allowance for losses arising from these two sources might perhaps increase the annual average to twenty-five.

Loss from the second cause is believed to be more important,

especially during the winter months when the observing conditions are frequently bad, but the amount is difficult to evaluate. It suggests, however, that thirty per year is a reasonable estimate of the frequency of novae in M 31.

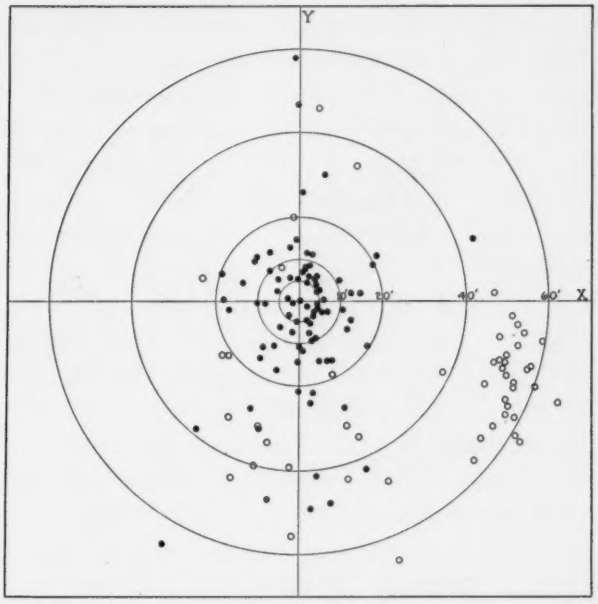


FIG. 9.—Distribution of novae and variables observed in M 31. Black discs refer to novae; open circles, to variable stars. X and Y are the major and minor axes of the nebula. The Y co-ordinates have been corrected to represent the plane of the spiral as perpendicular to the line of sight. The blank region to the left of $X = -30'$ is accounted for by lack of observations. The preponderance of negative signs among the larger numerical values of Y appears to be significant.

DISTRIBUTION OF NOVAE

The distribution of novae over the image of the nebula is shown on Plates IV, V, VII, where positions are marked. The positions are given diagrammatically in Figure 9, the Y co-ordinates being corrected to represent the spiral as perpendicular to the line of sight. Few novae have been found in the great rifts between the spiral arms, nor do they show any conspicuous relation to the numerous small dark markings. In general, the distribution of novae follows the distribution of luminosity in the nebula. The concentration in the nuclear

region is conspicuous, although it is noteworthy that only three novae besides S Andromedae have been found less than 4′ from the center. This does not appear to be wholly a result of limitations set by the dense nebulous background. The zone from 4′ to 8′ from the center is the most prolific, having produced twenty-eight novae, or about one-third of the total number observed. From 4′ outward, the total numbers in 5′ zones diminish steadily, and the numbers per unit area diminish even more rapidly, as may be seen in Table VIII.

Beyond the nuclear region there is an apparent asymmetry in the distribution in the sense that the observed novae are more numerous in the half of the nebula south following the major axis than in the north-preceding half. For $4Y > 15'$, there are only three novae in addition to the doubtful No. 39 in the north-preceding half, while in the south-following half there are eighteen besides the doubtful No. 26. When other variable stars are considered, the discrepancy is even greater—six objects as against thirty-one. In the north-preceding half, the nebula is less luminous, the rifts between the arms more pronounced, and, in general, the mottling due to dark markings is more conspicuous. This suggests a correlation with some form of obscuration, but the relation is not definitely indicated. The mean magnitude of the three novae in the north-preceding half, Nos. 27, 56, and 77, is 17.6 for the observed, and 16.95 for the computed maxima. These are somewhat fainter than the corresponding values for the novae in general, but the data are too limited for the differences to be regarded as significant.

Among the novae, as among the Cepheids, there is no definite relation between luminosity and distance from the nucleus. The data are shown in Table VIII.

These results indicate the absence of any considerable absorption by the unresolved luminous portions of the nebula, although obscuration by well-defined dark markings is conspicuous. The fact that the nucleus is sharply visible, although buried to a depth of many hundreds of parsecs of nebular material, points in the same direction.

The observational data bearing on the composition of the nuclear region are (1) the lack of absorption, (2) the infrequency of very bright giants ($M < -3.2$), (3) the appearance of novae, and (4) the characteristic dwarf solar-type spectrum with broad fuzzy lines. A

superficial interpretation of these data suggests a star cloud in which bright giants are rare, in contrast with the outer arms of the spiral, where giants are abundant. The Cepheids would then follow the distribution of the giants, while the novae would follow that of the fainter stars, or possibly that of the stars in general.

From the observed shapes of nebulae, however, Sir James Jeans has concluded that the nuclear regions of spirals, as well as the elliptical nebulae, must be gaseous.[1] The lack of resolution is thus

TABLE VIII

FREQUENCY OF NOVAE AT DIFFERENT DISTANCES FROM THE NUCLEUS

ZONE	NOVAE	MEAN DISTANCE	MEAN MAXIMUM	
			Obs.	Cal.
0′– 5′	12	4′.0	17.32	16.51
5 –10	25	7.2	17.04	16.31
10 –15	19	12.9	17.35	16.59
15 –30	13	19.5	17.03	16.34
30	13	44.5	17.38	16.69

accounted for, and since the densities are low—of the order of 10^{-21}, the lack of appreciable absorption is also explained. The novae and the spectrum raise difficulties, but these, according to Jeans, are not unsurmountable. The novae, for instance, might possibly be explained as due to the penetration into the gaseous region of stars from the outer portions. This would also account for the lower frequency of novae in highly resolved nebulae (e.g., M 33, N.G.C. 6822, and the Magellanic Clouds), where the unresolved regions are relatively small.

THE BRIGHT NON-VARIABLE STARS

No general investigation of the luminosity function of the giant stars in M 31 has been undertaken, but measures have been made of stars in the cluster N.G.C. 206, which appears to belong to the spiral. This is an open cluster in Region 4, about 41′ south preceding the nucleus along the major axis ($X = 40'.8$; $Y = +4'.4$). Its dimensions are about $6' \times 2'$, and the elongation is roughly north-south. About ninety stars are brighter than photographic magnitude 18.5, twelve

[1] *Astronomy and Cosmogony*, p. 336, 1928.

of which may be attributed to the galactic foreground. At the distance indicated by the Cepheids, the dimensions of the cluster would be of the order of 480×160 parsecs, and apparent magnitude 18.5 would correspond to absolute magnitude −3.7. There are no definite indications of stars brighter than −5.5, but at −5.0 they begin to be fairly numerous. Preliminary inspection of other fields in the nebula suggests the possibility of a few stars brighter than −6.0, but that large numbers do not appear until −5.0 is passed.

Very bright stars appear to be relatively more numerous in M 33, where, in general, they seem to be white or blue. In M 31 the brightest stars appear to have a slightly larger color-index, averaging perhaps +0.3 or +0.4 mag. The stars in N.G.C. 206 have been measured on Cramer Iso plates, exposed with the 100-inch reflector for two hours through a visual color-filter, the scale being established by three comparisons with the North Polar Sequence made with the 60-inch reflector. The various plates are not of the best quality; but the internal agreement is rather good, and the results probably indicate the order of the colors. The faintest photovisual magnitudes represent an extrapolation of about 1.5 mag. beyond the standard stars.

A plot of the color-indices against the photographic magnitudes is shown in Figure 10. The galactic latitude of the cluster, +22°, is only 6° less than that of the North Pole, hence the distribution of the colors among the foreground stars should approximate that among stars in the Polar Sequence, although the polar stars may be expected to average slightly redder. A probable lower limit to the color-indices of the foreground stars similar to that used in discussing the color-indices in M 33,[1] may be derived from these data. This limit has been indicated in the figure by a dotted line, and the numbers of stars above the line are roughly the numbers of foreground stars to be expected within the area. The area is too small to inspire any confidence in statistical conclusions, but the results suggest that red stars are not frequent among the brighter stars in the cluster and that the average color-index of the latter is about 0.35 mag.

Photovisual plates of other regions, notably Region 4, about 10′ south of N.G.C. 206, appear to be consistent with these conclusions, but the scales have not been sufficiently well established to permit

[1] *Mt. Wilson Contr.*, No. 310; *Astrophysical Journal*, **63**, 236, 1926.

numerical comparisons. The Cepheids, however, clearly have larger color-indices than the general average of the other stars in this region of the nebula, and appear to be approximately of the solar type. Great clouds of dark material occur in these outer regions, but the normal residuals from the period-luminosity curve of the Cepheids suggest that absorption, either general or selective, is not appreciable beyond the observed limits of these clouds.

No patches of luminous diffuse nebulosity have been found in M 31, although they appear, along with blue giants, in several of the

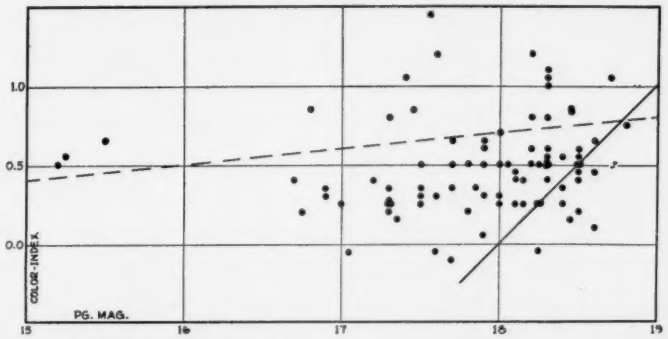


FIG. 10.—Color-indices of the brighter stars in N.G.C. 206, an open cluster in M 31. The diagram includes all stars down to 18.5 pg. mag. The dotted line indicates the probable lower limit of color-index among the foreground stars. The number of stars above this line is about the number of foreground stars to be expected. The data on color indices are complete to the full line with a slope of 45°; below this line, the photovisual magnitudes are incomplete.

most conspicuous of the late-type spiral and irregular nebulae. Diffuse nebulosities with emission spectra are the most easily identified, and these seem normally to accompany the characteristic of high apparent resolution. In general, such resolution implies an abundance of super-giant stars, and it is possible that the presence of emission nebulosities may indicate the relative frequencies of blue stars among these super-giants. The argument is rather speculative, but, since emission nebulosity is found only in highly resolved nebulae, it encourages the idea of a progressive tendency toward blueness among the brightest stars along the normal sequence of nebular types, such as is suggested by the comparison of color-indices in M 31 and 33.

Another suggestion arising from the comparison, namely, that brighter absolute photographic magnitudes are attained in the late-type spiral M 33 than in that of the intermediate type M 31, seems also to be an example of a more general relation. The evidence, scanty and rather ambiguous, is found in the slight systematic decrease along the sequence of nebular types, and in the difference between the luminosities of the brightest stars and the total luminosities of the nebulae in which they are involved. In an earlier discussion of such differences, a mean value was derived for all available nebulae, of whatever type, in which stars could be detected. A closer inspection of these data, supplemented by additional material, suggests the systematic effect above mentioned.[1]

MASS AND LUMINOSITY DENSITIES IN M 31

Some notion of the conditions within the nebula may be derived from rough estimates of the density both of mass and of luminosity. At a distance of 275,000 parsecs, the scale is $1' = 80$ parsecs, and the radius of the spiral, $80'$, is 6400 parsecs. By analogy with spirals of the same type seen edge on,[2] the semi-minor axis is assumed to be one-eighth of the radius, or 800 parsecs. The order of the volume can be approximated by supposing the figure to be generated by an isosceles triangle with an altitude four times the base, rotated around the base. This gives

$$V = \tfrac{2}{3}\pi \frac{(6400)^3}{8} = 6.9 \times 10^{10} \text{ cubic parsecs}.$$

The mass of the inner region, out to $2'.5$ from the nucleus, can be estimated on the assumption that the spectrographic rotation represents motions in circular orbits in a gravitational field. Then,[3] since

[1] *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926. No stars have been found in any of the Sa spirals, or in any of the elliptical nebulae except M 87 (N.G.C. 4486), where the difference between the nebula and the brightest stars is about 10 mag. For 14 Sb spirals, the average difference is about 9.6 mag., and for 12 Sc spirals, about 9.0.

[2] *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926.

[3] The formula Mass $\odot = 235 a v^2$, where a is the distance from the center in parsecs and v is the velocity of rotation in km/sec., is derived from the familiar formula for circular orbits in the solar system, Mass $= v^2 r k^{-2}$.

the velocity of rotation at 2′.5, or 200 parsecs, from the nucleus is about 72 km/sec.,

$$\text{Mass} = 235 \times 200 \times 72^2 \odot = 2.4 \times 10^8 \odot .$$

The total mass of the nebula follows from the arbitrary assumption that it is ten or twelve times that of the inner region. A value previously used, $3.5 \times 10^9 \odot$, is probably of the right general order.[1]

The absolute visual magnitude, -17.2, which follows from Holetschek's value for the apparent magnitude, 5.0, is probably a lower limit, but may be used for the rough calculations.

With these values, the mean luminosity density, in absolute visual magnitudes, is 9.9 per cubic parsec, and the mean mass density is about one sun per 20 cubic parsecs. Both densities are of the same order as those for the galactic system in the vicinity of the sun. In M 33,[2] the luminosity density is slightly brighter, 9.3 per cubic parsec, but the estimated mass density, 3 $\odot$ per cubic parsec, is about sixty times greater, and the discrepancy suggests the inadequacy of the methods used in determining the masses, especially in the case of M 33.

In terms of the sun's mass and luminosity, the relation

$$\text{Mass} = 5.5\,L ,$$

which holds in M 31, has a coefficient about twice that for the relation in the vicinity of the sun. The inaccuracy of the data, however, justifies only the conclusion that the coefficients are of the same order in two cases. The coefficient for the relation in M 33, 160, again suggests the inadequacy of the estimated mass of that spiral.

The central sphere, with a radius of 2′.5 = 200 parsecs, which appears to rotate like a rigid body, has a mass, as previously computed, of $2.4 \times 10^8 \odot$ and a volume of 3.4×10^7 cubic parsecs. The mean mass density is about 7 $\odot$ per cubic parsec, or 5×10^{-22} in c.g.s. units. If the central sphere is assumed to be 2 mag. fainter than the entire nebula, the absolute visual magnitude is -15.2 and the mean

[1] *Mt. Wilson Contr.*, No. 324; *Astrophysical Journal*, **64**, 321, 1926.

[2] *Mt. Wilson Contr.*, No. 310; *Astrophysical Journal*, **63**, 236, 1926. The radial component of the velocity of rotation of M 33 is derived from the difference between the radial velocities of the nucleus and of a patch of emission nebulosity in one of the arms. The velocity of the nucleus depends upon one spectrogram on a very small scale and is very uncertain.

luminosity density is 3.6 M per cubic parsec. The relation between mass and luminosity,

$$\text{Mass} = 2.3\,L,$$

is of the same order as for the nebula as a whole, but this result follows from the tacit assumption involved in estimating the whole mass, namely, that in the outer regions, the distribution of mass tends to follow the distribution of luminosity.

As the nucleus is approached, however, the mass density remains constant (according to the assumption of uniform angular rotation), while the luminosity density is observed to increase rapidly. The nucleus itself, on the shortest exposures, is sensibly round, with a diameter of about 3″, or 4 parsecs, and an apparent photographic magnitude about 14.0, or −9.2 on the absolute visual scale. Since the volume of the sphere is about 34 cubic parsecs, the mean luminosity density is of the order of −5.4 M per cubic parsec. The mass density, on the assumption of uniform angular rotation, is 5×10^{-22}, or 7 ⊙ per cubic parsec as given above.

The relation between mass and luminosity is now of the order

$$\text{Mass} = 0.001L,$$

which clearly does not represent the relation to be expected in a star or a cluster of stars giving a dwarf spectrum. Inaccuracies in the data will not seriously affect the order of the luminosity density, hence the uncertainties must be referred to the estimation of mass. If the latter approximates the proper order of magnitude, the mass-luminosity relation becomes significant and suggests that the nuclear material is in a gaseous state.

The nucleus of M 33 is very similar to that of M 31. The two show about the same spectral type, diameter, and absolute magnitude. The mean luminosity densities are not very different from those in the central regions of the more compact globular clusters. M 3, for instance, has a total absolute magnitude of −9.1, and the central sphere, with a diameter of 4 parsecs, is possibly 2 mag. fainter. The mean luminosity density of this inner region is therefore about −3.3 M per cubic parsec, or about one-seventh as bright as in the nuclei of the spirals. Nothing definite is known concerning the masses

of globular clusters, hence comparisons of mass densities cannot be made. The similarity in the luminosity densities suggests an analogy which is not encouraged by the mass-luminosity relation in the nuclei of the spirals.

A comparison of M 31 with the galactic system involves even greater uncertainties than those encountered above. Seares's[1] recent discussion of the galactic system, based upon star counts in the Selected Areas, suggests that it resembles a very late-type spiral, from 60,000 to 90,000 parsecs in diameter, in which the nucleus, 15,000–20,000 parsecs distant according to Shapley, is hidden from the earth by the obscuring clouds which form the great rift in the Milky Way. The sun is perhaps halfway out from the nucleus to the border, in one of the condensations represented by the local cluster, hence the density in our vicinity would be of the same general order as, or perhaps somewhat greater than, the mean density of the system. This lends some significance to the comparison of mass and luminosity densities in the vicinity of the sun with those for M 31.

If a diameter of 80,000 parsecs is assigned to the galactic system and a ratio of diameters of 1 to 10, the volume is of the order of 1.3×10^{13} cubic parsecs. The most plausible estimate of the mass is perhaps that suggested tentatively by Eddington,[2] based on the assumption that the observed motion of the system of the globular clusters is in reality a reflection of the rotation of the galaxy. Using Shapley's minimum value of the distance of the sun from the center of the system, he finds a mass of 2.7×10^{11} $\odot$. These figures lead to a mean mass density of one sun per 50 cubic parsecs as compared with one sun per 20 cubic parsecs in M 31.

Some notion of the order of magnitude of the total luminosity of the system can be derived by estimating the luminosity of the visible region and applying a reasonable correction for the invisible. Kapteyn's very generalized representation of stellar distribution out to star densities one one-hundredth that in the vicinity of the sun provides about the only data available. The numbers of stars to this limit (out to a distance of 8465 parsecs in the galactic plane) are

[1] *Mt. Wilson Contr.*, No. 347; *Astrophysical Journal*, **67**, 123, 1928.

[2] *Monthly Notices, R.A.S.*, **88**, 331, 1928.

equivalent to the numbers in a volume of 1.45×10^{10} cubic parsecs of unit density.[1] Since the unit density corresponds to $8.16M$ per cubic parsec, the absolute magnitude of Kapteyn's system is

$$8.16-2.5\log 1.45\times10^{10}=-17.24\,.$$

The larger system, with an assigned diameter of 80,000 parsecs, has a volume about twenty-seven times that of Kapteyn's, hence, if the latter were representative, it would be about 3.6 mag. fainter than the former. The great amount of obscuration along the galactic plane, especially between us and the central region, probably more than balances the influence of the local cluster and suggests that the magnitude difference should be materially increased. Possibly the round number $-22\,M$ may represent the luminosity of the galactic system within 2 or 3 mag. This leads to a mean luminosity density of $10.8\,M$ per cubic parsec as compared with $9.9\,M$ in M 31. The relation between mass and luminosity, Mass $=5.2\,L$, is also about the same as in the spiral.

These comparisons indicate a general similarity which further data are not likely to disprove. The outstanding discrepancy between the two systems is in their dimensions. According to the present figures, the galactic system is five or six times the diameter of the spiral, although the latter is the largest of the extra-galactic systems whose distances are reliably known. M 31, however, is in an intermediate state of concentration in the observed sequence of spiral forms, and could be spread through a much greater volume without losing the characteristics of an extra-galactic system.

Moreover, while the galactic system is very large, the outer regions, as viewed from a distant source, are very faint. Seares,[2] using a method independent of the spatial distribution of stars, finds that the surface brightness of our region, as viewed from a direction perpendicular to the galactic plane, is about 22.93 in visual magnitudes per square second of arc. Beyond a certain lower limit, this, of course, is independent of distance. Photographic magnitude 23.5 per square

[1] *Mt. Wilson Contr.*, No. 230; *Astrophysical Journal*, **55**, 302, 1922. See also Sir James Jeans, *Astronomy and Cosmogony*, p. 15, 1928, where Kapteyn's data are given in fuller detail.

[2] *Mt. Wilson Contr.*, No. 191; *Astrophysical Journal*, **52**, 162, 1920.

second of arc represents about the limiting faintness that can be registered by long exposures with reflectors of focal ratio 1 to 5, and this, corrected by a reasonable color-index, is of the same order as Seares's result. Viewed from a direction in the galactic plane, the surface brightness would be considerably greater, but it is improbable that photographs from a distant point would show the full dimensions suggested by the investigations of Seares and Shapley. This, together with the greater concentration in M 31, tends to reduce the apparent disparity in size between the two systems very materially. The galactic system must be considered as much larger than M 31, but the ratio is not greater than that between M 31 and other known extra-galactic systems.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
December 1928

DESCRIPTION OF PLATES

PLATE III

SOUTH PRECEDING END OF MESSIER 31

Plate by Duncan, August 24, 1925; 100-inch reflector, exposure 2 hrs. on Eastman 40 plate. Scale 1 mm = 15″.1. Top is north. The bright patch 34 mm to the right of the brightest star (B.D.+39°158) is a defect.

PLATE IV

MESSIER 31

Plate by Ritchey, 24-inch reflector at Yerkes Observatory, September 18, 1901. Scale 1 mm = 39″.1. Novae are indicated by crosses; variables, by lines and a V preceding the number. V1, 15, and 19 can be seen on the plate. The open cluster N.G.C. 206 is 3 mm below nova No. 46. Variables in Region 4, centered 13 mm to the right of N.G.C. 206, and novae near the nucleus are not marked.

PLATE V

CENTRAL REGION OF MESSIER 31

Plate by Duncan, 100-inch reflector, 9 hrs. exposure, September 16–17, 1920. Scale 1 mm = 16″.4. Novae are indicated by crosses, excepting Nos. 36 and 80, which are visible and enclosed in circles; variables, by a V preceding the number. V2, 3, 7, 9, and 13 are near maxima.

PLATE VI

VARIABLES IN MESSIER 31

Plates with 100-inch reflector; scale 1 mm = 8″.0. *Top.*—V1 near minimum, August 4, 1924. At maximum it equals the brighter of the two stars above and to the left. *Middle.*—V5 near maximum; V10 near minimum, August 5, 1924. The large object is M 32. *Bottom.*—V12 near maximum; V7 and 9 near minimum, July 30, 1924. The bright star is B.D.+40°151.

PLATE VII

MESSIER 31, REGION 4

From same negative as Plate III. Scale 1 mm = 14″.2. The patch 36 mm to the right of the brightest star, B.D.+39° 158, is a defect. The cross marked 40, a mistake for 46, indicates the only nova observed in this region. All other numbers refer to variables. V48 to the left of V25 should read V47. V40 is on the edge of a small compact cluster, of the same order of dimensions as a globular cluster. V23, 24, 28, 30, 34, 37, 38, 39, and 46 are near maxima.

PLATE VIII

VARIABLES IN REGION NO. 4

Plates with 100-inch reflector. Scale 1 mm = 6″.5. Right-hand plate, October 26, 1924; left-hand plate, August 24, 1925. Variation is conspicuous in V25, 26, 30, and 39.

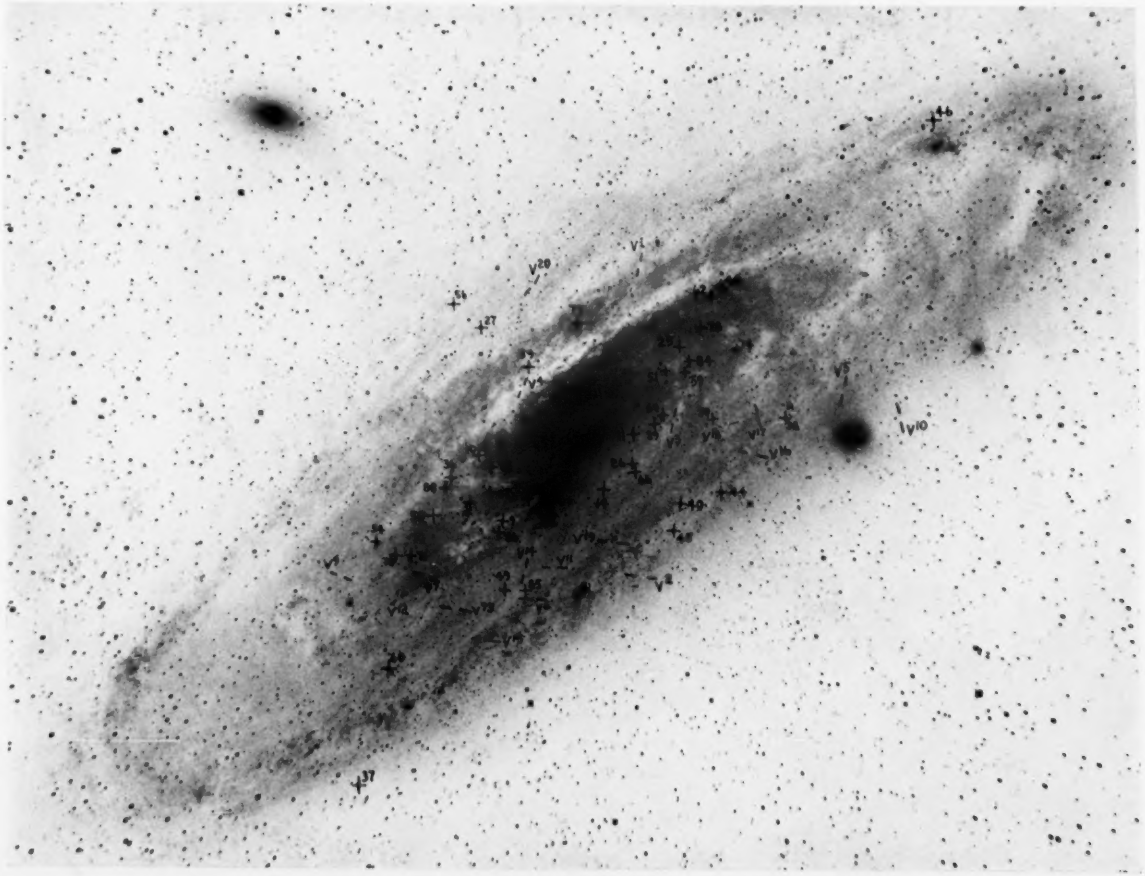
PLATE III

N

UNIV. OF MICH.

PLATE IV

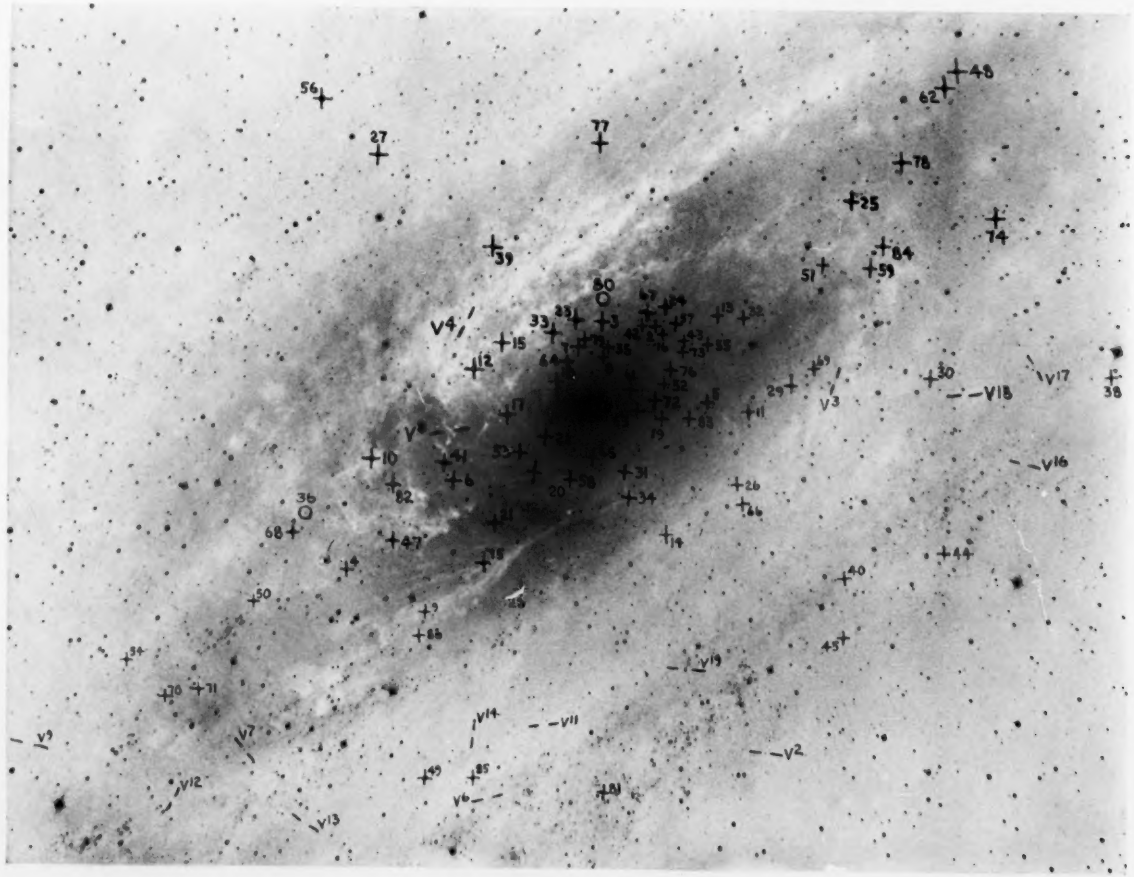
N



S

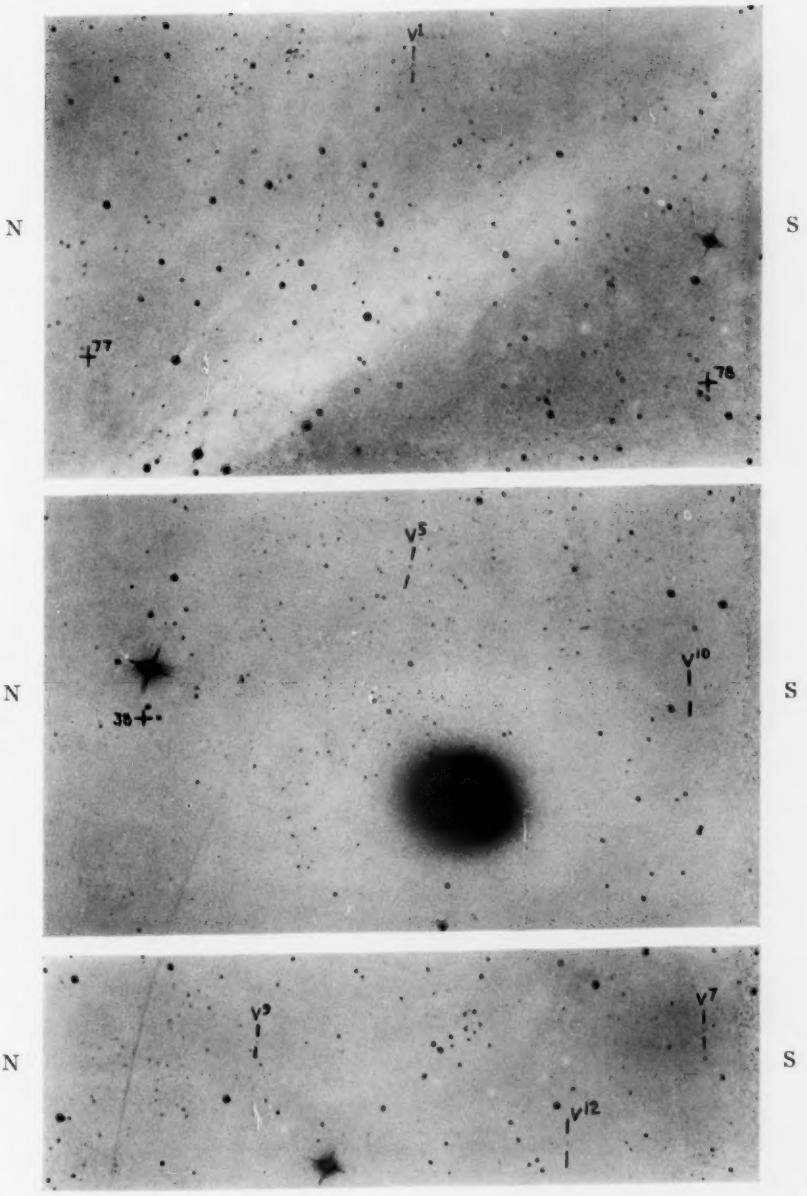
UNIV OF MIC

PLATE V



UNIV. OF MICH.

PLATE VI

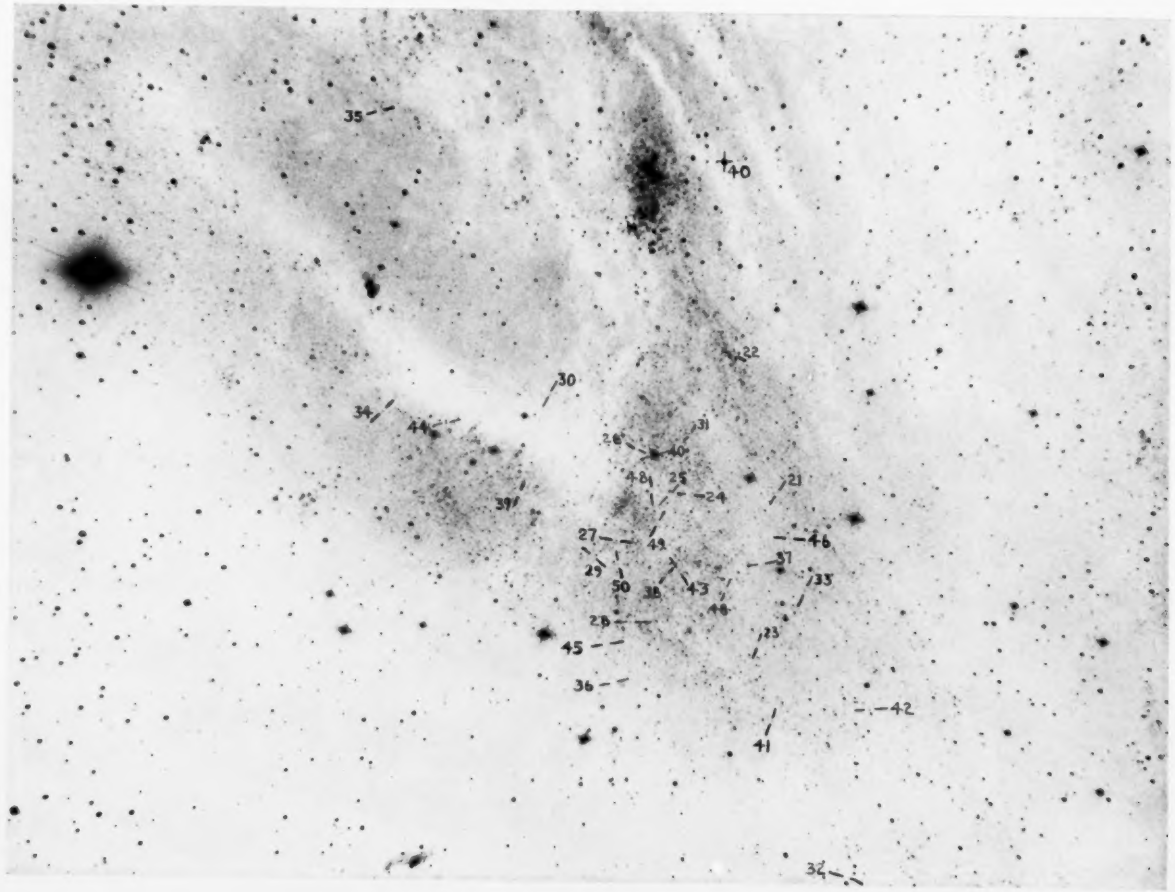


UNIV. OF MICH.

PLATE VII

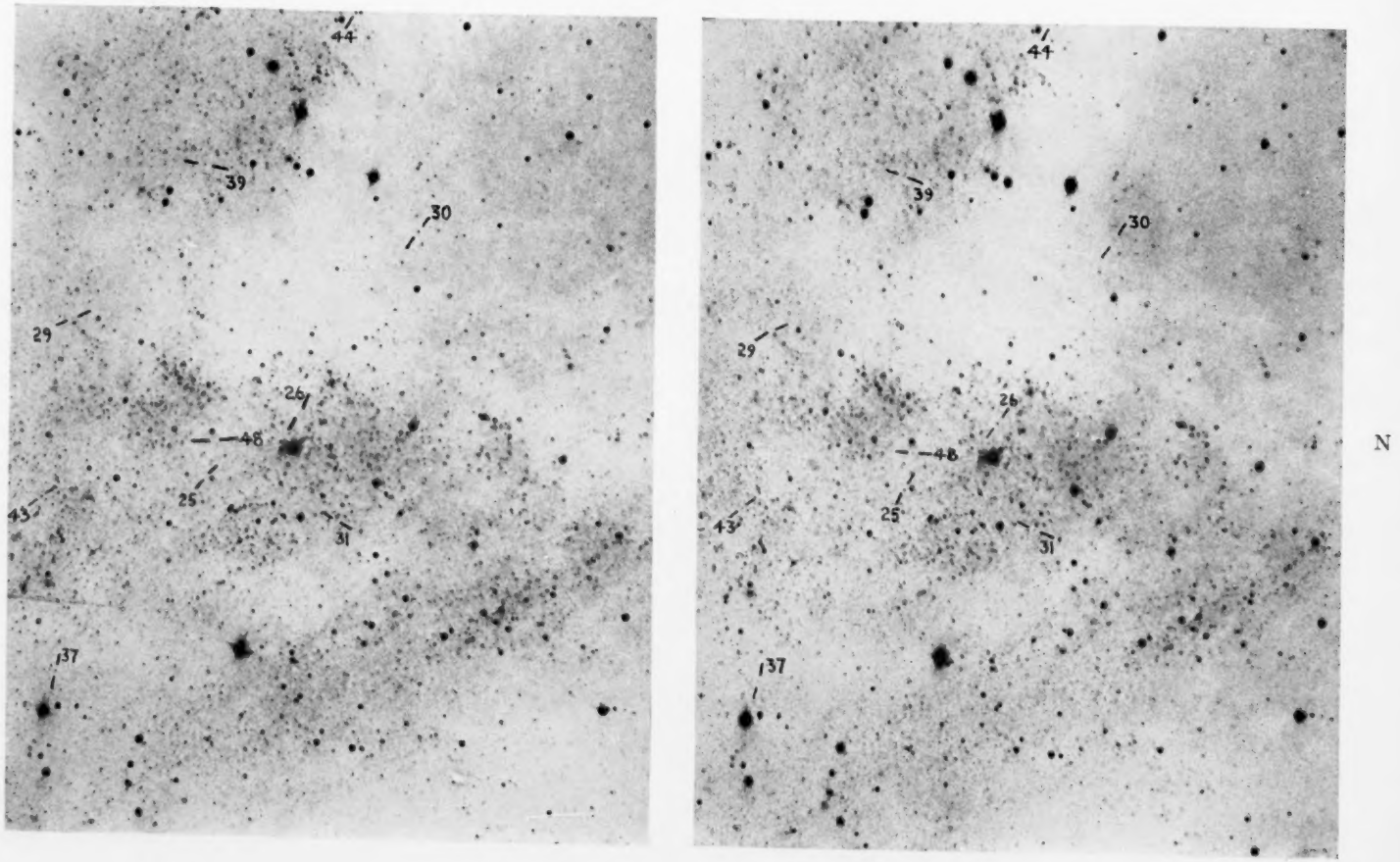
PLATE VII

N



UNIV. OF MICH.

PLATE VIII



UNIV. OF WICH.

ON THE HELIOCENTRIC VELOCITY OF METEORS

By C. HOFFMEISTER

ABSTRACT

Heliocentric velocities are found from observations of *daily variations* of shooting-stars which do *not* satisfy the hypothesis of a *cometary origin*. *Hyperbolic velocities* are *found* for the *majority* of meteors, which are about 2.4 times the earth's orbital speed, or *one full unit greater* than the *parabolic*. This must divide the field of meteoric astronomy into two streams: (1) those certainly cometary currents, and (2) those much *more numerous, originating in interstellar space.*

The values of heliocentric velocity are averaged from observations extending throughout the year.

It is found that the *mean velocity* undergoes rather large and irregular *changes annually*. There is, also, a smaller change with a period of a day. These facts lead to the deduction of three theorems (to be supported observationally in a future more extensive publication): (1) The smaller the zenith distance of the apex (the tangential point), the less the mean heliocentric velocity. (2) The mean heliocentric velocity is smaller as the share of motion among the apparent paths is larger. (3) The changes in mean heliocentric velocity vary inversely with the mean hourly number of meteors for the apex on the horizon.

In an investigation published in 1922[1] I have shown that it is impossible to satisfy the observations of the daily variation of shooting stars by the hypothesis of the general cometary origin of these bodies and that it is necessary to accept a heliocentric velocity much larger than that corresponding to elliptic or parabolic motions. The value of the velocity found from the curve of daily variation is about 2.4 in units of the earth's mean velocity, and is a full unit larger than the parabolic velocity. This means that aside from the certainly cometary currents most of the meteors move in strongly hyperbolic orbits and neither have their origin in the solar system nor show any relation to the comets, but come to us from interstellar space, as was noted long ago for most of the large fireballs by von Niessl and others. The whole phenomenon of shooting stars has, therefore, to be divided into two parts: the cometary and the generally much more significant interstellar component.

1. Being just now engaged in an extension of my former work, I think that it may be useful to show in how far these results have been confirmed by experience and by the criticisms of other astronomers. We may first consider an investigation by Fessenkoff and

[1] *Fragänzungshefte zu den Astronomische Nachrichten*, 4, No. 5, 1922.

Scigolev,[1] who made tests as to whether there is any relation between meteors in general and periodic comets. The result was negative. Fessenkoff points out that cometary meteors seem to be in the minority and that we must accept the conclusion that hyperbolic velocities occur for the majority of shooting stars. In his excellent book,[2] C. P. Olivier deals with the subject extensively, and reaches a decision in agreement, but finds it, with this view, necessary to test the result by further observations. The only serious objection has been made by E. Oepik,[3] concerning not the method itself, but the observational basis. He noted that the observer does not count masses of meteors, but luminosities, and that luminosity might be strongly influenced by differences of geocentric velocity so that the observed curve of daily variation is by no means the correct one. On applying these corrections he finds a curve, which leads to two solutions: either the mean heliocentric velocity of meteors may be about six times as large as that corresponding to the circular motion, or it may be equal to the parabolic or even less. He finds an additional condition, that most of the meteors must have direct motions. In an extended paper[4] I have shown the possibility of a physical theory of meteors, which does not lead to Oepik's conclusions, and I regard his criticism as refuted. The discussion[5] which followed my publication did not yield any points of view of particular interest, but as the physical theory of meteors rests on a very uncertain basis,[6] I wish to say that Oepik's conclusions also contain some contradictions in the much more certain field of celestial mechanics.

The first of the two possibilities admitted by Oepik agrees in principle with my results and differs from them only by the higher velocity, but Oepik does not assign to it much probability. As to the second one I must recall the following:

In my second publication[7] I have tried to derive the mean hyper-

[1] *Astronomische Nachrichten*, **220**, 227, 1923–1924.

[2] *Meteors*, pp. 194–198, Baltimore, 1925.

[3] *Astronomische Nachrichten*, **219**, 97, 1923.

[4] *Ibid.*, **221**, 353, 1924.

[5] *Ibid.*, **223**, 73 and 79, 1924–1925.

[6] Cf. my remarks in *Meteorologische Zeitschrift*, p. 464, 1927.

[7] *Astronomishe Nachrichten*, **221**, 373, 1924.

bolic velocity also from direct determinations, i.e., length of real path divided by number of seconds of duration. Besides this I have used a new method which depends only on the heights of disappearance and, therefore, is wholly independent from the estimates of duration. The observational data I took from the very large material of computed real paths of shooting stars published by Denning and also from a less extensive collection of computations of my own. Bright meteors had been excluded. The whole material was divided into two sections, by means of the apparent radiants; the "probably cometary" meteors and those "not known to be of cometary origin." The first group contained solely members of the great cometary currents. The result was as follows. Let 42 km/sec. be the velocity of "probably cometary" meteors, the velocity of the second group of meteors in km/sec. will be:

	a) From Direct Determinations	*b*) By the Method of Heights Weighted	Same Unweighted
Collection Denning (236 meteors).....	47·5	47·2	48·0
Collection Hoffmeister (13 meteors).....	53·7	48·7	53·5

This is a direct proof for hyperbolic velocity. The reason for these values being smaller than that one found from daily variation is that this kind of observation gives a preference to the bright meteors, and to the slow ones. I was able to show that the velocity is larger in proportion as the observations have a less accidental character.[1] Hence the values found above can only be taken as the lower limit of mean velocity. This will be for meteors "not known to be of cometary origin" certainly > 1.2, if we assign the value 1 to the "probably cometary" meteors.

Applied to Oepik's second possibility this means that if we give to the non-cometary meteors the highest velocity admitted by him, 42 km/sec., the velocity of the cometary meteors must be < 35 km/sec. The orbits of Perseids and Lyrids would be ellipses of small eccentricity, which contradicts certain results on this subject.

[1] At this time I call attention to the fact that photographic observations will give a preference to bright and slow meteors in a still higher degree and that, therefore, mean values of height, velocity, etc., derived from photographic observations may differ widely from the correct average values.

Finally, the prevalence of direct motion is not in agreement with the hypothesis of cometary origin and also would require a new astronomical theory of meteors different from that given by Schiaparelli.

2. It must be said that no really serious objection has been found up to date against the interstellar origin of most of the meteors. A theory, free from contradictions, can be constructed. Schiaparelli's system, per contra, requires the whole phenomenon to be divided into two groups of different masses: the fireballs with hyperbolic and the shooting stars with elliptic or parabolic orbits, a rather improbable postulate. This difficulty now disappears, but we must not say that another one, the separation of shooting stars into the cometary and the interstellar component, is taking its place. It must be remembered that the division of a uniform visual phenomenon into parts in respect to origin or cosmical position is by no means exceptional. For example, we see in the field of a telescope the fixed stars intermingled with a small number of minor planets, but optically both are "stars." Besides this, one of the two alternatives has to be accepted if we do not deny the cometary origin of the Perseids or the interstellar source of large meteors.[1]

3. The theory of daily variation has been developed independently by the author and by Miss N. Staude at Leningrad.[2] A critical test was later made by Hepperger.[3] It is not to be expected that serious mistakes or omissions occurred.

The theory has up to the present been applied to four collections of observations. The following table contains the derived values of mean heliocentric velocity in units of the velocity of circular motion.

Observer	c
Hoffmeister	2.40
Schmidt	2.28
Coulvier-Gravier	2.43
Heybrock	2.49

[1] The objections to be made to the recently published investigation of Dr. Willard C. Fisher (*Harvard Circular*, No. 331) I shall point out elsewhere. Indeed, I regard the interstellar origin of most large meteors as one of the best-known facts in meteoric astronomy.

[2] *Astronomische Nachrichten*, **218**, 155, 1923.

[3] *Sitzungsberichte der Akad. d. Wiss., zu Wien, Math.—Nat. Klasse*, **132**, Part II*a*, 9 und 10, 1923.

The average is 2.40. It is computed from observations distributed over the whole year, only the Lyrids, the Perseids, and the Geminids of December having been excluded.

In view of these numbers no further doubt of the reliability of the result as to the interstellar origin of most of the meteors can be possible. We must, however, avoid overestimating the accuracy of the numbers, for there are some sources of systematic errors. The preference for slow meteors mentioned above will cause the velocity to be found too high. Four influences counteract this error: (*a*) the observer is more attentive when many meteors become visible; (*b*) the state of sky may be generally better in the later hours of the night; (*c*) the velocity is found too small, if there are several groups with different velocities, as will be explained later on; (*d*) the preference for slow meteors also is extended to the cometary component, partly compensating for the error. Besides this, the preference for slow meteors is in reality not as large as the observations of real paths seem to indicate. It must not be overlooked that only those meteors could be used in the investigation mentioned above for which velocities were known. This requires the duration to be estimated, which is generally only possible if the duration is much longer than the average value for small meteors. The direct proof of hyperbolic velocity therefore gains a still higher weight.

The preference for slow (and bright) meteors must be greater as the atmospheric conditions are less favorable. It is therefore expected to be shown strongest by observations from large towns. Indeed, the observations by Heybrock at Frankfurt-am-Main give the highest velocity, but the difference is not large. Really the conditions must have been rather bad at Frankfurt, for the mean hourly number observed by Heybrock is only 6.7 compared with 9.2 observed by the author at Sonneberg under good conditions. Much larger is the difference of velocity found by the author from his observations at Baltimore, but the real reason for this discordance may be that at Baltimore conditions for observations often become less favorable after midnight, the transparency being diminished by haze. The Heybrock observations show that the error in velocity arising from the preference for slow meteors cannot be large.

Nevertheless, the possibility that the value 2.4 is a little too high

must be admitted. If it were possible to take account of all errors we might find that the velocity derived by von Niessl for fireballs, about 2.0 or 2.2, is correct, but we cannot regard these values as accurate or be certain that the mean velocity of interstellar shooting stars ought to be exactly the same as that of large meteors.

4. The values of the heliocentric velocity used here are average values derived from observations extending throughout all the year. But I found that the mean velocity undergoes rather large changes of an irregular kind in the course of the year. These changes are the same when derived from groups of observations of different origin. Besides this, there exists a much smaller change of mean heliocentric velocity with a period of a day. I shall give here three theorems on these subjects, but without supporting them by observational facts. This will be done in a more extensive publication now in preparation.

THEOREM I. *The smaller the zenith distance of the meteoric apex, the smaller the mean heliocentric velocity becomes (distribution effect).*— The cause of this phenomenon is that there are components of very different velocity active at the same time. It is easily to be seen that the apparent radiants of slow, that means in this case cometary, meteors will be much more concentrated toward the apex than those of fast meteors. So that the hemisphere of the earth containing the radiant point will receive relatively many meteors of slow heliocentric velocity, and the average velocity will be diminished. In order to demonstrate the effect of velocity distribution I shall derive it from an assumed case. Let us suppose the phenomenon to be composed of two groups with the heliocentric velocities $c=1/\sqrt{2}$ and $c=3.0$, respectively, the earth's mean velocity being taken as the unit. The mean number of meteors per hour at the zenith distance of the radiant $z=90°$ may be 4.0 for each group. From the formulae given in my first publication the corresponding curves of daily variation are found as given on page 165.

The sums represent the resulting curve of daily variation, as it would be observed under the supposed conditions. And if we now calculate backward the velocity from the resulting curve with restriction to observational conditions of mean geographic latitudes we find:

From curve branch $z=50°-90°$ ..... $c=1.89$
From curve branch $z=90°-130°$ ..... $c=1.97$

Conditions at the equator are more favorable, especially at the time of solstice when observations covering the whole curve from $z = 20°–160°$ can be made in a single night. But also in the observations now available there are some indications of the effect of distribution.

At the same time it is shown that the resulting velocity is about 0.3 smaller than the average of the component velocities. The reason is that the mean of two variation curves does not correspond to the mean of velocities, for $\frac{\delta N}{\delta z} \cdot \frac{dz}{dc}$ (where N is the hourly number of meteors), is not a linear function of c.

z	Group		Sum
	1	2	
0°	10.11	6.87	16.98
10	10.01	6.83	16.84
20	9.71	6.70	16.41
30	9.19	6.48	15.67
40	8.51	6.18	14.69
50	7.68	5.81	13.49
60	6.78	5.39	12.17
70	5.83	4.93	10.76
80	4.89	4.47	9.36
90	4.00	4.00	8.00
100	3.19	3.55	6.74
110	2.48	3.14	5.62
120	1.88	2.76	4.64
130	1.41	2.44	3.85
140	1.05	2.17	3.22
150	0.79	1.96	2.75
160	0.63	1.82	2.45
170	0.55	1.74	2.29
180	0.53	1.72	2.25

If we make the unit of abscissae of the curve of variation $= 15°$,

$$\Sigma N = \int_0^{\pi} (fz) \cdot dz$$

will be the whole number of meteors seen at the equator at the times of the solstice while the apex rises from $z = 180°–0°$. Here $f(z)$ means the expression given as formulae (9) in my first publication. We find:

For $k = 4.0$ $c = \sqrt{2}$ ..... $\Sigma N = 55.7$
For $k = 4.0$ $c = 3.0$ ..... $\Sigma N = 50.0$

And if we now compute the mean velocity with ΣN as weights, we get 2.16. The mean heliocentric velocity derived from the combined curves of daily variation therefore always stays below the true value, if there exists an actual distribution of velocities.

A twofold importance is to be attributed to the effect of distribution. First, its existence, when reliably shown by observations, will be a new independent proof for the existence of an interstellar group of meteors, for with respect to the curve of daily variation it is quite inadmissible to make the velocity of the second group less than the parabolic. Second, the effect of distribution, though small and difficult to detect, will perhaps give valuable aid in the numerical separation of the two components, if probable suppositions as to the distribution functions of velocities can be made. But it seems that there are still better ways to come to this end.

THEOREM II. *The mean heliocentric velocity is smaller, as the share of regular motion among the apparent paths is larger (the effect of radiation).*—To make this clear I must state that I have developed some methods, depending on the theory of probability and permitting me to find out which portion of the radiants, when determined in the ordinary way, are due to accident, which portion of all meteors, therefore, may be regarded as "sporadic," i.e., the directions of motions being distributed in an irregular way without indication of radiation in currents. I found large changes in the course of the year of the systematic and sporadic portions respectively, showing the correlation with the velocity-curve mentioned in the second theorem. The interpretation will be that meteors emanating from a real radiant are mostly of cometary character and sporadic meteors are mostly of interstellar origin.

THEOREM III. *The changes of mean heliocentric velocity "c" and of mean hourly number of meteors "k," referred to the zenith distance of apex "z" = 90°, are always opposite to each other.*—The observational data available now do not contradict the presumption that this theorem might be exactly fulfilled, although this cannot be expected; at least the approximation is very good. The close correlation between normal numbers of meteors and heliocentric velocity is well represented by the presumption that the interstellar share may be constant throughout the year, the cometary component being

of variable strength, thus synchronously increasing the number and diminishing the mean velocity in different degrees for different periods.

The second and third theorems include another possibility for separating the components and for detecting their relative changes. The observations now at our disposal will be sufficient to answer these questions.

SONNEBERG, THÜRINGEN, STERNWARTE
August 1928

REVIEWS

Analyse periodischer Vorgaenge. By KARL STUMPFF. "Sammlung geophysikalischer Schriften." Gebr. Borntraeger, 1927. Pp. x+189. Plates 1, Figs. 41. M. 14.40.

There have been published during the last decade several important books on practical mathematics (e.g., those by Runge, Wittaker-Robinson, von Sanden, Lindow, etc.) as well as more or less extensive tables facilitating harmonic analysis (Pollak may be mentioned, for example) and the applications of special functions. Having these books and tables at hand, any practical scientist, even without advanced mathematical education, can nowadays perform the most delicate calculations and draw from his crude observational data all the necessary conclusions. The excellent books mentioned above cover the most important chapters of practical mathematics, such as the classical least-squares method, smoothing of data, interpolation, statistical methods, Fourier analysis, etc. Unfortunately these works do not include an adequate treatment of the study of the periodograms or the methods facilitating the search for hidden periodicities. It seems hardly necessary to emphasize how important those methods are for modern astrophysics, geophysics, and meteorology. In most questions of classical astronomy (including here tidal analysis), the superposed periods are known a priori, and all that is necessary is to disentangle them and to find their respective amplitudes. This problem is solved in an easy and practical way by Fourier analysis. But the modern scientist is not in such a favorable position; the periods hidden in his data are not known a priori, and it is his task to discover and study them.

Dr. Stumpff's book seems to fill the above-mentioned gap in the literature on practical mathematics. Dr. Stumpff himself has made some original contributions to this branch of science which renders his book of special interest. One-third of the book under review is of rather introductory character, dealing briefly with the methods of smoothing of data, correlation, and more completely with classical Fourier analysis. Very complete and interesting is the third chapter describing the methods of detecting the periodicities based on transformed harmonic analysis (those by Turner, Sommerfeld, and Prince Galitzin). Especially useful are the methods based on the numerical calculation of differential quotients and

finite differences. All these originate from Oppheim's mathematically very elegant, though not practical, ideas, which were extended and later transformed by Bruns, Hirayama, and Dale. So far as I am aware, those methods have never been set forth in a systematic and comparative form. They are very elegant but too complicated and not suitable for practical use.

The really practical method for finding hidden periodicities is due to Schuster, who based himself on Buys-Ballot's theorem on the properties of the sums of m equidistant observational data arranged in matrix form. Schuster's periodogram method is explained by Dr. Stumpff with thoroughness and clearness and with due references to the recent papers by Craig and by Turner. Dr. Stumpff emphasizes the importance of the "phase-diagram," extending and completing its theory and giving a new definition of the periodogram. All his considerations are certainly interesting and deserve full attention; but what we really need now is the extension and transformation of Schuster's method according to the principle of practicality first. From this point of view the chapters on the analysis of discontinuous series of observations are of special interest. It is not generally known that the periodic gaps in the data (due, for example, to seasonal or climatic causes) introduce spurious periods into the periodogram. Instead of one spectral line (adopting the well-known Schuster analogy) we get a spectral series, with a limit corresponding to the period equal to zero. The last chapter deals with the apparatus used for the analysis of the periodogram, such as W. Ball's "harp," Prince Galitzin's analysater based on the principle of resonance, and A. E. Douglass' optical periodograph. The description of the optical periodograph by Stumpff and the possible applications of stroboscope and movie camera completes the book. The references are fairly complete, but for some unknown cause important analysis of RV Tauri by periodograms by van der Bilt is not mentioned at all.

The lack of practical examples certainly is one of the defects of the book. As an example, Dr. Stumpff analyzes Greenwich observations of the moon's right ascensions, seeking the periodicities in the difference between the observed and the computed values. This example is certainly very clear and instructive, but it seems to be too simple as compared with most cases which are to be studied by the method of the periodogram. Take, for instance, the case of semi-regular variables, where no a priori periods are known and where the investigator is fog bound among the enormous accumulation of non-homogeneous and incomplete data. Solar phenomena and earthquake data present other examples of the complex-

ity of problems to be solved, and show the urgent need for purely practical methods in the search for periodicities. From this point of view the present development of methods of dealing with the periodograph is quite insufficient. Since 1906, when Schuster finally formulated his method, only small contributions have been made to this new branch of practical mathematics. This certainly cannot satisfy the needs of modern physical science.

Dr. Stumpff's book is both useful and important, and is to be recommended to anyone who is interested in the study of periodic or simply cyclic phenomena.

B. P. GERASIMOVIČ

Ergebnisse der exakten Naturwissenschaften. Edited by the Schriftleitung der *Naturwissenschaften* (Berlin), JULIUS SPRINGER. V (1926), 329; illustrations 103; bound, M. 22.50. VI (1927), 378; illustrations 85; M. 20.

The weekly journal of science, *Die Naturwissenschaften*, follows for the German scientific public the model of that great record of scientific progress *Nature* founded sixty years ago by Norman Lockyer. Under the able editorship of Arnold Berliner with the collaboration of Hans Spemann, *Die Naturwissenschaften* is now in its seventeenth year, and its pages give an increasing attention to astrophysics.

From their contact with the research workers in the field of natural science, the editors have been in a position to bring to publication articles which were too extensive for the pages of a weekly journal. They have therefore issued, through the press of the publisher of their journal, Julius Springer, Berlin, a series of important supplementary volumes, of which seven have thus far been issued. We shall notice at this time the fifth and sixth volumes of the series.

Volume V contains nine of these essays, practically all of them of astrophysical interest: E. Schoenberg, "Ueber die Strahlung der Planeten" (46 pp.); P. Seliger, "Das photographische Messverfahren" (48 pp.); A. Wegener, "Ergebnisse der dynamischen Meteorologie" (28 pp.); N. Bjerrum, "Die elektrischen Kraefte zwischen den Ionen und ihre Wirkungen" (20 pp.); P. Pringsheim, "Lichtelektrische Ionisierung von Gasen" (18 pp.); G. Kirsch, "Atomzertruemmerung" (26 pp.); K. W. F. Kohlrausch, "Der experimentelle Beweis fuer den statistischen Charakter des radioaktiven Zerfallsgesetzes" (20 pp.); E. Pietsch, "Gasabsorption unter dem Einfluss der elektrischen Entladung—*Clean Up*—und verwandte

Erscheinungen" (53 pp.); H. Kallmann and H. Mark, "Der Comptonsche Streuprozess" (58 pp.).

The first paper gives a careful discussion of the radiation of the planets including the results of oppositions of Mars as late as 1924. The article is illustrated by copies of R. W. Wood's photographs of Saturn in infra-red, yellow, violet, and ultra-violet. Dr. P. Pringsheim's paper is of direct astrophysical bearing. The next paper deals with the breaking-up of atoms. Each of the papers is accompanied by an excellent bibliography and numerous diagrams where needed.

We find in Volume VI eleven papers as follows: H. Vogt, "Der innere Aufbau und die Entwicklung der Sterne" (26 pp.); E. Freundlich, "Die Energiequellen der Sterne" (16 pp.); W. Steinhaus, "Ueber unsere Kenntnis von der Natur der ferromagnetischen Erscheinungen und von den magnetischen Eigenschaften der Stoffe" (30 pp.); H. Sponer, "Optische Bestimmung. Dissoziationswaerme von Gasen" (28 pp.); H. Cassel, "Zur Kenntnis des adsorbierten Aggregatzustandes" (19 pp.); W. Braunbek, "Zustandsgleichung und Zustandsbegrenzung des festen Koerpers" (30 pp.); W. Orthmann, "Kritische Arbeiten zur elektrostatischen Theorie der starken Elektrolyte" (45 pp.); K. F. Bonhoeffer, "Ueber die Eigenschaften der freien Wasserstoffatome" (30 pp.); E. Brodhun, "Die Entwicklung der Photometrie in diesem Jahrhundert" (47 pp.); P. Seliger, "Das photographische Messverfahren" (53 pp.); Ida Noddak and Walter Noddak, "Das Rhenium" (40 pp.).

We may suggest that our readers will find the first and second articles of obvious interest. Dr. Bonhoeffer's discussion of the free atoms of hydrogen is thorough and extensive; and Dr. Brodhun contributes a careful statement of the progress of photometry since the beginning of the present century with illustrations of new apparatus and methods. The paper on "Das Rhenium," by its discoverers, Herr and Frau Noddak, is the most comprehensive contribution regarding this element which has come to our attention.

Die seltenen Erden vom Standpunkte des Atombaues. Von G. HEVESY. Berlin: Julius Springer, 1927. 8vo. Pp. 140. Plates 8. Bound, M. 10.20.

A number of excellent texts have been published on the rare earths dealing with the occurrence, extraction, and separation involved in the study of this unique group of elements. The various compounds and the

preparation of the free metals have been thoroughly reviewed. However, there has been none of the type written by Professor Hevesy.

It is natural that, in the last decade, the interest in the theoretical aspects of the relationship of the rare earth to other elements, and that which they bear to each other, should be greatly stimulated by the work of Bohr and others on atomic structure. These contributions to theoretical chemistry have been applied with most interesting results, and this monograph is one of the first to deal with these facts in an authoritative manner.

The book is divided into two parts, the first of which deals with the rare earths and atomic theory. In it there is a discussion on the position of the rare earths in the periodic table, on their chemical behavior, and on the contribution to our knowledge from the recent spectroscopic work of the group. Part II gives a rather brief account of the chemical properties and occurrence of the rare earths. This includes a description of the preparation of the metals and of typical compounds. Methods of both qualitative and quantitative analysis are taken up as well as the most important separations. The dependence of crystal structure of compounds and the distribution of the earths in nature on ionic sizes are the final topics.

Professor Hevesy has given a masterly summary of recent advances, and in so doing has written a book that should be on the shelves of every inorganic chemist. It should stimulate interest in the rare earths. It makes no pretensions to being a comprehensive treatise but does succeed in presenting what its title promises.

It can be obtained well printed on excellent paper and in a good binding.

L. F. YNTEMA

Amateur Telescope Making. Edited by ALBERT G. INGALLS. Second Edition. New York: Scientific American Publishing Co., 1928. 12mo. Pp. xii+285. 147 Illustrations. $3.00.

Enlarged to nearly three times its original size when first published in 1926, this revised edition covers in an eminently satisfactory manner the field suggested by the title. Contributions to this edition by Hale, Hastings, Ritchey, Shapley, Elihu Thomson, and others further enrich the volume in interest and value.

S. B. B.